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ICE FORMATION ON AIRCRAFT AND ITS PREVENTION.

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PART I.

The formation of ice upon airplanes has presented such a menace to safety and to the maintenance of scheduled trips, particularly in the airmail service, as well as to any future commercial use of aircraft, that a research * into the causes and prevention of the ice deposition has seemed highly important. The study was undertaken in the hope that it would lead to a practical means of protecting the airplanes from the ice hazard during their flights through the ice forming regions.

Water occurs in the atmosphere in two forms, namely vapor and droplet. The subject therefore naturally divides itself into two parts.

At the inception of this work, a study of the bibliography of the subject indicated that in the opinion of many the vapor

* This work was done in the Department of Physics at Cornell University, under the direction of its Committee on Aeronautic Research: F. Bedell, Chairman, W. C. Geer, and C. C. Murdock, and was supported by a grant from the Daniel Guggenheim Fund for the Promotion of Aeronautics. A first publication has been issued as Technical Note of the National Advisory Committee for Aeronautics No. 345, entitled "The Prevention of the Ice Hazard on Airplanes," by W. C. Geer and M. Scott.

present in the atmosphere is largely responsible for the ice hazard encountered by aircraft. In particular, the reader is referred to a report on the subject to the Canadian Research Council by H. T. Barnes,[1] in which the formation of ice upon airplanes from vapor is presented and in some measure discussed. In part I of the present paper the possibility of a plane taking on ice from vapor is examined.

Consider an airfoil section moving with velocity, q, through a region in which the water vapor density is ρ and the corresponding water vapor pressure, p. The vapor molecules in their random motions bombard the airfoil surface. If the molecules of water reaching the surface sublime upon it, they form a snow-like type of ice called frost. During sublimation these molecules yield to the surface their heat of sublimation or the excess energy which they possess due to their rapid motions. This process of diffusion of the vapor molecules upon the surface should continue as long as the water vapor pressure in the atmosphere exceeds the saturation vapor pressure of the ice forming upon the airfoil surface.

The heat carried to the airfoil surface by the subliming molecules must be dissipated, otherwise the temperature would continue to rise until the ice melted. This dissipation is accomplished principally by the process of forced convection.

Quantitative calculations as to the amount of ice deposit to be expected and the general form of the deposition have been carried out. The details are presented below. It is found that the maximum deposit should come at the leading edge and that the deposit should decrease rapidly toward the trailing edge. The maximum rate of deposit to be expected with a maximum of supersaturation known to occur in the atmosphere is low and should not prove dangerous in an hour's flight through such a region.

It is assumed initially that the surface of the airfoil is a perfect heat insulator and that all the heat of sublimation is dissipated by forced convection. When the conductivity of the airfoil surface is considered, its effect is found to be

[1] Barnes, "Adhesion of Ice Research," Nat. Res. Council, Canada, 1929.

transient, lasting in the case of an aluminum surface about fifteen minutes.

In calculating the initial rate of deposit of the frost, the following equations are employed:

$$\mu = \rho q - \frac{1}{3}\lambda u \frac{\partial \rho}{\partial y}, \tag{1}$$

$$\mu = k \frac{d\rho}{dy}, \tag{2}$$

$$\frac{d\rho}{dt} = k \frac{d^2\rho}{dy^2}, \tag{3}$$

wherein μ is the mass of water vapor molecules crossing a unit area in direction of positive y per unit of time, ρ the mass density of vapor molecules, q the velocity of flow of gas in positive y direction, λ the mean free path of the vapor molecules, u the mean speed of the molecules, k the diffusion constant for the vapor molecules in air.

The first equation is taken from Meyer's [2] theory of diffusion of gases, the second serves merely to define the diffusion constant k, and the third is the equation of continuity for the vapor molecules.

The vapor molecules diffuse toward the airfoil surface during deposition. Their general movement may be thought to take place along tubes of diffusion which are normal to the air flow tubes. Figure 1 illustrates these tubes for a strut of usual section as well as for one infinitesimally thin. These tubes of diffusion may be considered fixed with respect to the air and hence to approach the airfoil with a velocity equal to the relative wind.

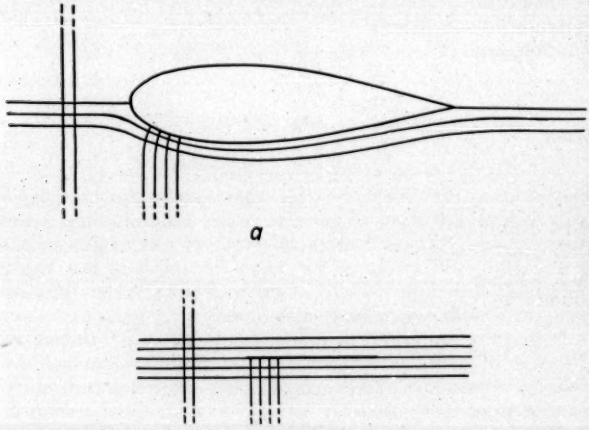
Before any given diffusion tube reaches the airfoil it is infinite in extent but upon arriving at the leading edge of the airfoil may be considered to be divided into halves. For each of these halves, the following boundary conditions obtain:

$$\begin{aligned} \rho &= \rho_i, \quad & y &= 0, \\ \rho &= \rho_0, \quad & t &= 0, \end{aligned} \tag{4}$$

[2] Jeans, "The Dynamical Theory of Gases," p. 309.

where ρ_i is the vapor density in equilibrium with ice, and the zero of ordinate is taken at the surface of the airfoil; ρ_0 is the vapor density in the air and the time is counted from the instant the diffusion tube reaches the leading edge of the strut.

FIG. 1.



Tubes of air flow and vapor diffusion.

In case of the strut Fig. 1 (*a*), each half of the diffusion tube is telescoped upon itself somewhat as the strut moves past but this does not account for any additional collection of vapor molecules by the sweeping process of the airfoil section for the air carrying the vapor molecules is swept back as well, and it is an accepted experimental fact that in such cases there is no differential separation of vapor molecules from the air.

It is true that the orderly movement of such diffusion tubes, at least close to the airfoil, is imaginary but since the boundary layer theory is in an unsettled state, the assumption made regarding these tubes seems best. Possibly the vapor should be considered to diffuse up the tube to the outer

surface of the boundary layer and then through this layer of continuously varying velocity. In this case the assumptions as made would apply properly to the diffusion process up to the outer surface of the layer and the effect of the boundary layer might then be neglected.

The diffusion of the vapor along any tube is expressed by equation (3) and the boundary conditions (4). To obtain a solution, consider the problem:

$$\frac{d\rho'}{dt} = k\frac{d^2\rho'}{dy^2}, \qquad \rho' = 0, \quad y = 0, \qquad \rho' = \rho_0', \quad t = 0.$$

Then

$$\rho' = \frac{2\rho_0'}{\sqrt{\pi}}\left[\frac{y}{2\sqrt{kt}} - \frac{y^3}{3(2\sqrt{kt})^3} + \cdots\right].^3 \tag{5}$$

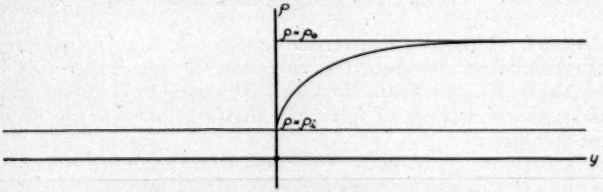
Now if ρ' is chosen so that $\rho' = \rho - \rho_i$, it follows that, since $\rho' = 0$ at $y = 0$, $\rho = \rho_i$ at $y = 0$, and that, since $\rho' = \rho_0'$ at $t = 0$, $\rho_0' = \rho_0 - \rho_i$. The solution of equation (3) and boundary condition (4) is therefore:

$$\rho' = \rho - \rho_i = \frac{2\rho_0 - \rho_i}{\sqrt{\pi}}\left[\frac{y}{2\sqrt{kt}} - \frac{y^3}{3(2\sqrt{kt})^3} + \cdots\right],$$

$$\rho = \rho_i + \frac{2\rho_0 - \rho_i}{\sqrt{\pi}}\left[\frac{y}{2\sqrt{kt}} \cdots\right]. \tag{6}$$

Equation (6) is represented in Fig. 2.

FIG. 2.



Diagrammatic sketch of the solution of the diffusion problem.

In equation (1), q, the velocity of flow of the gas in positive y direction is always zero from considerations already

[3] Byerly, "Fourier's Series and Spherical Harmonics," p. 85.

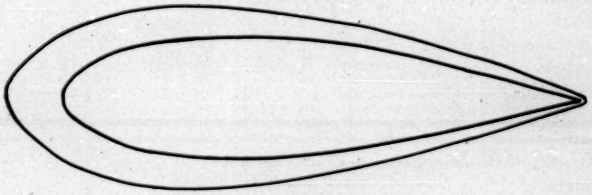
given. Combining equations (1), (2) and (6), the following is obtained:

$$\mu = \frac{1}{3}\lambda u \frac{d\rho}{dy} = k\frac{d\rho}{dy} = \frac{2k(\rho_0 - \rho_i)}{\sqrt{\pi}} \frac{1}{2\sqrt{kt}} \quad \text{at} \quad y = 0. \qquad (7)$$

Equation (6) has been differentiated term by term and the substitution $y = 0$ made. μ is the rate of sublimation at $y = 0$ or, as we have assumed, at the surface of the airfoil.

As a practical example, let the velocity of an airplane be taken as 5000 cm./sec. and the longitudinal dimension of a strut as 15 cm. It is possible then by use of equation (7) to tabulate values of t and μ and hence obtain the distribution of the initial rate of deposition along the strut surface. This method assumes that the diffusion tube moves back along the strut surface with uniform velocity equal to that of the

FIG. 3.



A sketch of the initial rate of frost deposition.

airplane. This is of course not true but it is not considered worthwhile to introduce the variation of velocity as this is relatively small. Table I shows corresponding values of t and μ based upon $k = .198$ while Fig. 3 presents a sketch of the formation.

TABLE I.

t sec.	Position on Strut.	μ gm./cm.^{3} sec.
.00005	Leading edge	2.73 $(\rho_0 - \rho_i)$
.0001	"	1.91
.0006	"	.79
.0012	"	.55
.0018	"	.45
.0024	"	.39
.0030	Trailing edge	.35

So far in this treatment the formation has been termed initial. The reason for this is that as sublimation continues, the deposit is not an amorphous layer but a prismatic crystalline growth as the wind tunnel tests to be described show. It is clear therefore that the above calculation cannot give an accurate description of the shape of the frost deposit but only an over all idea of the general form.

It has already been pointed out that changes in the deposition may occur in the boundary layer.

Also it has been implicitly implied that the heat of sublimation is perfectly dissipated. This is undoubtedly true initially but as the frost deposit grows the process cannot be accurately followed.

Equation (7) may be written

$$\mu = \text{const.}\ (\rho_0 - \rho_i), \tag{7a}$$

and expresses the fact that the rate of deposit is always proportional to the excess of vapor density in the air over that in equilibrium with the frost or ice upon the airfoil surface. It is clear then that the rate of formation is limited by the upper limit of the value of ρ_0, the water vapor density occurring in the atmosphere. A second limitation, as already mentioned, is that imposed by the ability of the heat of sublimation to be dissipated.

At this point the assumption is introduced that the surface of the airfoil is a perfect heat insulator and that all the energy is dissipated by forced convection. The rate of heat transfer is expressed by:[4]

$$H_e = \text{const.}\ \Delta T \tag{8}$$

wherein H_e is the rate of heat transfer in cal./cm.^{2} sec. and ΔT the difference in temperature between the surface and the main air stream.

In order to obtain numerical values the example upon which Table I was based is continued. The limitation of the rate of deposit is now considered for a point back somewhat from the stagnant line and for which the const. of equation (7a) equals 2 (see Table I). Due to scarcity of data on forced convection on airfoils, there is doubt regarding the

[4] Inter. Crit. Tables, Vol. 5, p. 235.

value of the const. in equation (8) but work by the author [5] indicates that for the part of the airfoil selected by the constant, 2, of equation (7a), this constant should be approximately .031.

Table II is now prepared. Column 1 contains a selected range of values of H_c, column 2 gives the values of ΔT calculated by eq. (8), column 3 gives the values of μ; it is obtained by dividing the numbers of column 1 by the heat of sublimation of water vapor (700 cal./gm.). Column 4 shows the excess of vapor density in the air over that in equilibrium with ice and is calculated by eq. (7a).

TABLE II.

$\frac{H_c}{\text{cal.}/\text{cm.}^2\,\text{sec.}}$	$\frac{\Delta T}{°\,\text{C.}}$	$\frac{\mu}{\text{gm.}/\text{cm.}^2\,\text{sec.}}$	$\frac{\rho_0 - \rho_i}{\text{gm.}/\text{cm.}^3}$
.01	.3	.000014	.000007
.025	.8	.000035	.000017
.05	1.6	.00007	.000035
.1	3.2	.00014	.00007
.25	8	.00035	.00017

ΔT is the difference in temperature between the surface of the airfoil and the air. This can never exceed the temperature difference between the air and 0° C., otherwise the frost would melt. If, for example, the air temperature is − 10° C., then the maximum value of ΔT for frost deposition is 10° C. This temperature difference determines the rate of heat dissipation and hence the rate of sublimation. If ρ_0, the vapor density in the air, has the correct value to make $\rho_0 - \rho_i$ equal to that given in column 4 (Table II), the deposit may be said to be "full," that is, the vapor density supplies vapor for sublimation just fast enough for the process of forced convection to dissipate the heat of sublimation. [If ρ_0 were larger than this, the sublimation would begin at a rate so fast that the heat of sublimation could not be dissipated and the frost would melt or cease to form. It is conceivable that in this case of high ρ_0 the frost would become very smooth and hard due to formation close to the melting point.]

[5] See part III.

Equation (7*a*) shows that deposition occurs whenever the vapor density in the air exceeds that in equilibrium with the frost upon the surface. Table III presents values of the relative humidity which must be exceeded at different air temperatures for sublimation to take place.

TABLE III.

Temp. of Surface °C.	% Humidity.
0	100
− 2	98.5
− 4	96.5
− 6	95
− 8	93
−10	91.5
−12	90
−14	88.5
−16	87.0

The limitation of the rate of deposit by forced convection occurs when the surface temperature reaches 0° C. The value of ρ_0 required for this is then that corresponding to a "full" deposition. The saturations or rather the ratios of the value of ρ_0 required for a "full" deposit to the vapor density in equilibrium with water at the air temperature may now be tabulated (see Table IV) assuming 0° C. as the surface temperature.

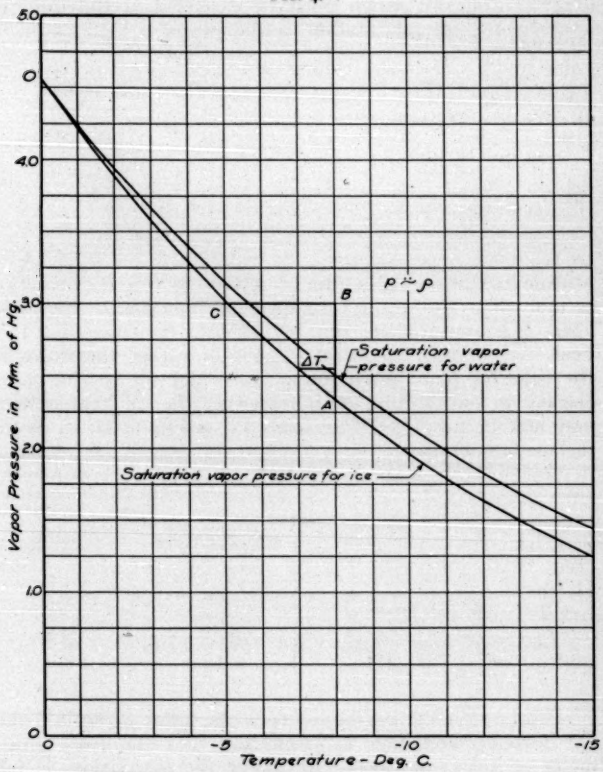
TABLE IV.

Air Temp. °C.	ΔT °C.	ρ_0 (Required for Full Deposit) $\frac{\text{gm.}}{\text{cm.}^3} \cdot 10^{-9}$.	ρ (Saturation over Water at Air Temp.) $\frac{\text{gm.}}{\text{cm.}^3} \cdot 10^{-9}$.	$\frac{\rho_0 \text{ required}}{\rho \text{ saturation}}$
− .3	.3	11.85	4.405	2.7
− .8	.8	21.85	3.828	5.7
−1.6	1.6	39.85	3.006	13.3
−3.2	3.2	74.85	1.820	41.2
−8	8	174.8	.375	466.0

If ρ_0, or the saturation, exceeds the value given in Table IV, forced convection is insufficient and the frost either melts or ceases to form. If ρ_0, or the saturation, is less than the value given in Table IV the formation takes place at a rate less than that allowed by forced convection. As the air temperature decreases the value of ρ_0 required for full deposit increases very rapidly. The values given in this

table apply strictly only when the surface temperature is 0° C. but are approximate for lower temperatures. For

FIG. 4.



Vapor pressure curves for water and ice.

example, if the saturation is 2.7 and the air temperature — .3° C., the surface of the airfoil will rise to 0° C. as the frost forms. If the air temperature were, say — 10° C.,

then a saturation of 2.7 would produce a temperature rise in the airfoil surface slightly greater than .3°.

Since supersaturations of more than 4 have never been observed and since experiments by C. T. R. Wilson [6] lead us to believe that higher supersaturations than this are unobtainable in air, the rate of formation is limited by this value and given as approximately .000035 gm./cm.^{2} sec. (see Tables II and IV). Since this rate of deposition is allowable at any air temperature below $-$ .8° C., it is clear that in general ρ_0, not forced convection, serves to limit the process.

The rate indicated could scarcely be expected to yield a deposit greater than 5 mm./hr. near the leading edge and this could in no wise be considered dangerous for an airplane.

The significance of these relations can be clearly outlined by reference to the curve in Fig. 4. Suppose the air temperature is $-$ 8° C. and the vapor pressure is 3.0. Then $p_0 - p_i$ is indicated by AB. The rate of deposit is given by Eq. (7a). The temperature of the surface rises because of the heat of sublimation until ΔT becomes sufficiently large for the forced convection to take care of the heat arising from the sublimation. Table II shows that the difference $(\rho_0 - \rho_i)$ and hence $(p_0 - p_i)$, which is approximately proportional to it, is so great for a "full" ice deposit that the limitation in deposit by forced convection never comes for $\rho_0 < 4.847.10^{-6}$, the value at 0° C. That is, forced convection is always more than sufficient to care for the heat of sublimation as long as $\rho_0 < 4.847.10^{-6}$, the value at 0° C. When, however, ρ_0 exceeds this value and reaches such values as given in Table II, the deposit is limited by the ability of forced convection to take away the heat.

The same may be stated by combining (7a) and (8) by multiplying (7a) by the heat of sublimation, thus:

$$700\,\mu = 2(\rho_0 - \rho_i)700 = .031\Delta T,$$

where $\rho_0 - \rho_i$ has its value limited by forced convection.

$$\frac{\rho_0 - \rho_i}{\Delta T} = \frac{.031}{1400} = .0000222$$

which is much larger than the slope of the $\rho - T$ curve.

[6] Wilson, *Phil. Trans. Roy. Soc.*, A, Vol. 189, p. 265, 192, p. 403.

Let it now be assumed that the surface is conducting and let the effect of this conductivity be examined. The effect must be simply to add to the amount of heat carried away from the surface and hence to allow larger rates of deposit. The problem can be abstracted as follows: (See Fig. 5.)

FIG. 5.

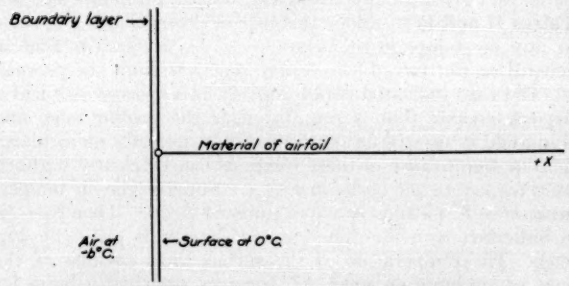


Diagram of the heat flow problem at the airfoil surface as abstracted.

Assume an air temperature of $-b°$ C. The material of the airfoil would then be everywhere at this temperature before deposit of ice begins. After deposition starts, let the rate be such as to raise the temperature of the surface to 0° C. The heat problem is then

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}, \qquad \begin{matrix} u = 0, & x = 0, \\ u = -b, & t = 0, \end{matrix} \tag{9}$$

and the solution is

$$u = \frac{-2b}{\sqrt{\pi}}\left[\frac{x}{2a\sqrt{t}} - \frac{x^3}{3(2a\sqrt{t})^3} + \cdots\right].^{7} \tag{10}$$

Then

$$\frac{\partial u}{\partial x} = \frac{-b}{\sqrt{\pi}a\sqrt{t}} \quad \text{at} \quad x = 0$$

and

$$H \text{ (conduction)} = k_c \frac{\partial u}{\partial x} t = \frac{k_c b}{\sqrt{\pi}a}\sqrt{t}, \tag{11}$$

[7] Byerly, Fourier's Series and Spherical Harmonics. Loc. cit.

where k_c is the conductivity constant for the material. Since the material of the surfaces before the deposit would have been at air temperature, the temperature difference $\Delta T = b$. Eq. (11) becomes:

$$H \text{ (conduction)} = \frac{k_c \Delta T}{\sqrt{\pi a}} \sqrt{t}, \tag{12}$$

whereas,

$$H \text{ (forced convection)} = .031 \Delta T \times t \quad \text{from eq. (8).}$$

Table IV gives a comparison of these two heats for different materials for a given value of $\Delta T = 3°$ C. The conclusion is that only the initial deposit can be affected, for, even for an aluminum airfoil, the rate of deposit is increased appreciably for only the first fifteen minutes. Thereafter the rate is practically independent of the conductivity and this would become increasingly true because of the layer of ice on the surface as Table V shows.

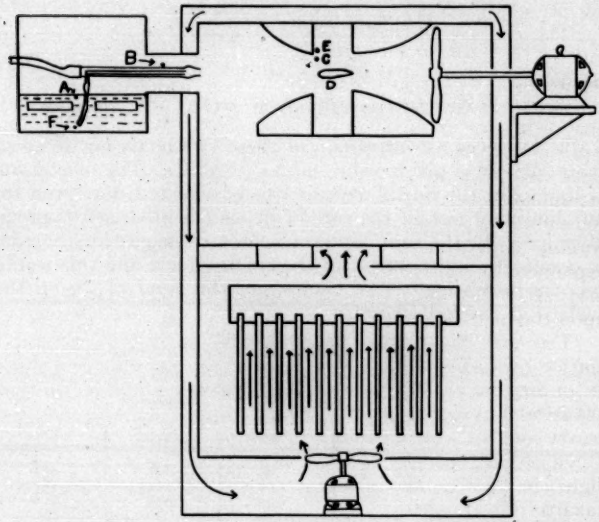
TABLE V.

$\frac{\Delta T = 3° \text{ C.}}{t}$ Sec.	$H_{\text{forced convection.}}$	$H_{\text{conductivity.}}$		
		Al.	Asbestos.	Ice.
1	.093	.54	.007	.051
9	.83	1.62	.021	.151
100	9.3	5.40	.070	.510
900	83	16.20	—	—

A wind tunnel was constructed for the purpose of studying the formation of ice as well as means of its prevention or removal. The requirements for studying the deposition of frost are that the humidity of the test chamber be controllable and variable over a wide range of values including supersaturations. Since the walls of the tunnel as well as the airfoil provide surface for condensation of the vapor, it is evident that water vapor must be supplied at a rapid rate to replace that condensed as frost. In order to secure supersaturations it is necessary that all hygroscopic particles be prevented from entering the tunnel and that those present be eliminated. These considerations led to the design of a

small wind tunnel of 7″ x 7″ cross section with a 3″ circular throat. The diagrammatic sketch is shown in Fig. 6.

FIG. 6.



The wind tunnel equipped for study of ice formation either from water vapor or from water droplets.

The water vapor is supplied by an evaporator which contains two electric heating units for maintaining the water at a given temperature. There were three thermo-junctions for measurement of temperature, one in the water at *A*, one in the channel at *B* and the third in the air stream at *C*.

The theory of operation is as follows. The surface of the water is maintained at temperature $t°$ C. and the vapor pressure in equilibrium with the flat water surface is thereby determined. The vapor begins to diffuse into the tunnel and air stream. At first it may meet nuclei, condense upon them and hence form drops, but these drops are thrown to the

walls at the corners by the failure of the viscosity force and the normal pressure force to hold them in the tubes of flow. By this process the air of the tunnel is cleared of nuclei and the vapor pressure is established in equilibrium with the surface of the water.

The deposit of frost forms upon the strut section at *D* and its character may be studied. Good formations have been obtained in 35 minutes of the form already suggested by theoretical considerations (see Fig. 3) but evidently crystalline and of a flowery pyramidal type. The general character is qualitatively as predicted and the rate of formation is as close as could be expected to that calculated, although no attempt at accurate quantitative results has been made. Actually the deposition falls off more rapidly toward the trailing edge than Fig. 3 indicates and may possibly be explained by the presence of the transitional layer, which may serve to hold the vapor and sift it more thoroughly upon the airfoil than would otherwise be the case.

The conclusion to be drawn from this study of the formation of ice from vapor is that frost formed upon the parts of an airplane by the diffusion and condensation of vapor is beautiful but not at all dangerous. The flowery growth is crusty and, in a sense, hard but has practically no resistance to vibration; changes of wind direction such as are met in flight, in yaw, pitch, and roll would certainly prevent any hazard from this source.

The model airfoil section was treated with goose grease [8] and other substances but none showed any tendency to prevent the deposition save that a heat insulating layer tended to retard the formation initially probably due to a retarded lowering of the temperature of the airfoil after its introduction into the tunnel. These tests were discontinued at this point in order to proceed with the study of the formation of ice from droplets.

Before leaving the subject it seems worthwhile in light of the work herein presented to reply to certain views offered by Barnes in his report to the National Research Council of Canada. In this report he suggested that the condensation of the vapor takes place upon the airplane surfaces cooled

[8] Suggested by Barnes, loc. cit.

differentially with respect to the air by radiation, that the airplane surfaces are further cooled by evaporation of dew and that the temperature of the airplane surfaces therefore during deposition is below that of the air.

It seems impossible to explain ice formation by supposing that the airplane surfaces are cooled by radiation faster than the neighboring air and hence that condensation takes place upon them. If this differential radiation effect were present, it would be much more than engulfed by the heat exchanges taking place by forced convection.

It seems impossible also to explain ice formation by supposing the plane surfaces cooled in region A, receiving dew in a warm, more humid region B, losing dew by evaporation in a dryer region C, hence being cooled all the more and prepared to receive ice in a humid region D when the surface temperature is below freezing. This argument neglects two factors: (1) forced convection which tends to keep the surfaces at the same temperature as the air; and (2) the fact that in region B the surface receives heat equal to hx where h is the heat of vaporation of water at the temperature and x the number of grams of dew deposited; and in region C it loses heat equal to hx' where x' is the number of grams of dew evaporated and in ice region D takes on a maximum number of grams of ice equal to hx/h' where h' is the heat of sublimation of vapor. Since h' exceeds h, the amount of ice would always be less than the amount of dew and may be neglected.

Simple calculations on forced convection show this process to be quite effective in maintaining an equality of temperature of the surfaces and the air. The conclusion is therefore reached that this effect accounts for all appreciable heat or energy interchanges. It follows that since sublimation of vapor upon the plane gives to the surfaces the heat of sublimation, the temperature of the surfaces must be always above that of the air for a steady deposit.

PART II.

Experience gained from the operation of air craft indicates that there are two general meteorological conditions under which ice may form upon their exposed surfaces during flight. One of these conditions is that of a temperature inversion

with an ice or glaze storm and the other consists in a region of mist, fog or cloud with the temperature below the freezing point of water. These conditions, together with the types and characteristics of the ice formed in each case, are discussed at length in technical notes[9] of the National Advisory Committee for Aeronautics. A presentation of the weather conditions under which ice is found to be a hazard to the practical operation of airplanes is given by Smith.[10] The possibility of the formation of ice from vapor becoming a hazard for aircraft has been discussed in part I.

By a temperature inversion is meant a meteorological condition such that the highest temperature occurs not at the surface of the earth, as is considered normal, but at a surface above that of the earth. The surface at which the temperature gradient reverses sign may be termed the temperature inversion surface. Above this temperature inversion surface, the normal temperature lapse rate obtains while below it the temperature decreases down to the surface of the earth. If a snow storm is present above the inversion and if the temperature at the inversion surface is above the melting point, then it is to be expected that the snow flakes, in falling through this region, will become melted and thereby converted into rain drops. These latter continue earthward passing into colder regions where they may freeze upon any object with which they come into contact.

If the surface of maximum temperature is sufficiently high above the earth and if the temperature gradient below is fairly steep, the rain drops may freeze before reaching the earth's surface. The precipitation is then designated as sleet. In this case there are actually in the atmosphere three types of precipitation: near the earth, sleet; above this, freezing rain or glaze; and above the inversion plane, snow.

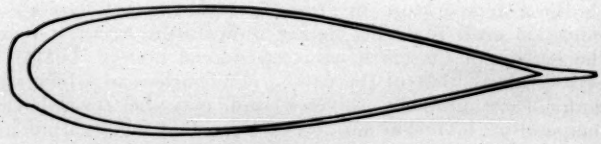
Practical experience indicates that sleet does not attach itself to the airplane surfaces and hence does not seriously interfere with the operation of the airplane through it. Also dry snow storms present no ice hazard for aircraft. In the

[9] N. A. C. A Technical Note 293, "The Formation of Ice upon Exposed Parts of an Airplane in Flight," Carroll and McAvoy. N. A. C. A. Technical Note 313, "The Formation of Ice Upon Airplanes in Flight," Carroll and McAvoy.

[10] Smith, Monthly Weather Review, Vol. 57, Dec. 1929, pp. 503–506.

case of the temperature inversion then, there remains only a thin layer below the inversion surface in which an airplane may not be operated without ice hazard. In this layer the exposed surfaces of any aircraft rapidly become glazed with ice which causes malformation of the lifting and control surfaces and gradually overloads the craft. The general form of the ice is as shown in Fig. 7. Frequently the wings are coated top and bottom and icicles extend from the trailing edge.

FIG. 7.



Typical glaze ice formation.

The other general conditions under which the formation of ice upon aircraft presents a hazard is due to the mixing on a large scale of air currents or layers of air of different temperature and of different moisture content. The result in the atmosphere is a huge region in which there are minute droplets in the form of fog, mist or cloud.[11] If the temperature of this region is below the freezing point of water, then it presents a definite ice hazard. Recording instruments [12] placed on air mail planes operated between New York and Cleveland on regular schedule provide data to show that under these conditions ice formation of a hazardous character occurs when the humidity is 90 per cent. or more and when the temperature ranges from 0° C. to as low as − 23° C. The pilots have in all cases reported that the ice accumulated when they were flying in fog, mist or cloud. When such a condition is well developed, a biplane with bracing wires can operate for only a few minutes (15 min. to 20 min.) due to vibrations set up as a result of the ice accretion in the wires,

[11] See Smith, loc. cit.

[12] These data were obtained by the U. S. Weather Bureau through its stations at Hadley Field and the Cleveland Municipal Airport. The instruments were carried by National Air Transport planes. Andrus, *Monthly Weather Review*, Vol. 58, Jan. 1930, pp. 22–24.

struts, radio mast, engine and even in the wings. The general form of the ice is shown in Fig. 8.

Ice formations upon miniature strut sections have been studied in the laboratory under controlled conditions simulating those in the atmosphere. The wind tunnel used in these experiments on ice formed from water droplets was adapted from the one described in part I of this paper. An aspirator, equipped with an electric heating element to prevent freezing, was mounted in the channel at *B* (see Fig. 6). An additional thermojunction was added at *E* just out of the airstream in order that a better control of conditions might be possible. And a vessel for water was located within the evaporator. No temperature control was incorporated into this vessel but the temperature was maintained as desired by the addition of ice. Thermojunction *A* was mounted at the intake of the aspirator at *F*.

The miniature struts were mounted in a holder at *D* and the formations under varying conditions were studied. The factors under control were temperature of air, temperature of water and air speed. Evidently when the air temperature is slightly below 0° C. and the water temperature above 0° C., the case corresponds to that of a temperature inversion.

FIG. 8.

Typical fog ice formation. Note the cloudy portion.

Under these conditions the ice is observed to be hard and clear with many icicles lying back on the strut or trailing to the rear. (See Fig. 7.) As the temperature of the water approaches 0° C., the ice formation becomes milky and assumes a crescent shape fringed with fine prismatic crystals. (See Fig. 8.) As the air temperature becomes lower the ice becomes more cloudy, apparently coarser grained and finally snow white. It is hard ice and its adhesion to an aluminum or mahogany airfoil surface is firm.

The air speed apparently has little effect upon the character of the ice formation.

The explanation of the whiteness of the ice is probably that usually given, namely the inclusion of air. As the temperature of the air stream and the droplets introduced into it is lowered, the degree of undercooling is intensified. The rapidity of freezing increases and the amount of air included should be expected to be larger. In this way the ice becomes at first milky, then almost snowy white. However the hardness of the ice, qualitatively at least, is not greatly reduced. The grain of the ice appears to be much coarser at the lower temperatures of formation.

A long series of tests was carried out to observe the effect of treating the airfoil surface with waxes, greases and oils. The procedure was to bring the air in the wind tunnel to some predetermined temperature, usually between $-1.5°$ C. and $-10°$ C., insert the airfoil, then introduce the rain or mist. As the airfoil would be at room temperature before insertion, it required about three minutes to come to the temperature of the air stream. In most of the tests, therefore, the mist was introduced three minutes after the wind was started. The apparatus was adjusted to produce a rate of ice formation comparable with that met under typical ice forming conditions in the atmosphere. This was based upon actual flight tests.[13]

The aluminum or mahogany airfoil was covered with a water resistant film whose effectiveness was qualitatively tested by holding the treated surface under a stream of water. If the surface upon withdrawal from the water presented itself free of water drops or films, it was considered worthy of wind tunnel tests under ice forming conditions. When the airfoil was mounted in the airchannel, it was initially at room temperature. If the wind were started and the mist introduced immediately, therefore, observations could be made of the behavior of the surface toward the mist as its temperature gradually approached that of the air stream. Although approximately seventy-five tests of this type were conducted in the wind tunnel, the results were all quite similar.

[13] These flight tests were made in coöperation with the National Air Transport at Cleveland, Ohio.

In the case of all surfaces which are water resistant at room temperature, the water droplets impinging upon the airfoil near the leading edge flatten into a thin layer and are then observed to spring up into spherical drops and race rapidly over the surface of the airfoil, leaving it for the most part at the trailing edge. Their movement is very rapid and resembles that of water droplets on a hot plate in the so-called spheroidal condition save that their movement is always definitely rearward. As the temperature of the airfoil decreases, the velocity of movement of the drops is less. The velocity continues to decrease until there is no motion. Freezing then takes place. In case the air stream temperature is close to the freezing point, say $-1.5°$ C., the oncoming drops maintain a film of liquid water over the ice which is forming upon the airfoil surface. This liquid water tends to flow rearward and occasionally, as described above, drops spring up and are observed to be driven rearward; sometimes they escape into the airstream, sometimes they contribute to the formation of a layer of ice or an icicle upon the surface of the airfoil or to an icicle at the trailing edge.

At still lower airstream temperatures, no film of liquid water is observed after the motion of the drops has ceased. Ice forms almost unnoticed near the leading edge. Its cross-section has the shape of a crescent symmetrically located over the stagnant line or leading edge of the strut. It varies from milky to snowy white and apparently from a finer grain to a considerably coarser ice. To the solid crescent or cap there develops a fringe of scattered prismatic crystals standing up into the wind, as already described (see Fig. 8).

The phenomena above described were observed in the cases of all the water resistant surfaces tested. The particular treatment of the airfoil surface did not seem to affect the result materially. It appears, therefore, that the ice formation is controlled principally by factors other than the nature of the surface.

The following calculation indicates that in a glaze storm, under the condition of temperature inversion, ice should be expected to be accumulated only when the temperature is close to the freezing point.

Suppose a drop to be falling at its terminal velocity

through air of uniform temperature. It is cooled by forced convection. The conductivity of the drop is considered sufficiently large so that temperature differences within the drop may be neglected. The heat lost by forced convection is given by the formula, [14]

$$H_c = .00309\Delta T \times t, \tag{13}$$

where H_c is the heat lost by the entire drop in calories, ΔT is the temperature difference between the drop and the air, and t is the time in seconds. Then

$$.00309(T_d - T_a)dt = \frac{\pi}{6}a^3\rho s dT_d,$$

where T_d = temperature of drop, T_a = temperature of air, ρ = density, and s = specific heat of drop. For this example, a drop has been chosen, whose diameter is .1 cm. and whose terminal velocity is 400 cm./sec. Upon integration the above equation becomes

$$\log \frac{T_d - T_a}{(T_d - T_a)_0} = -kt, \tag{14}$$

where $(T_d - T_a)_0$ is the initial temperature difference between the drop and the air and k equals 6.2. Table VI gives for chosen values of t the distance the drop has fallen and the ratio of the temperature difference between the drop and the air to its initial value.

TABLE VI.

t sec.	d (cm.)	$\frac{T_d - T_a}{(T_d - T_a)_0}$
.1	40	.24
.2	80	.06
.3	120	.01

The conclusion is reached that the falling drop almost immediately assumes the temperature of the air through which it falls.

Now consider a drop falling through a temperature gradient below a temperature inversion surface. The heat

[14] Int. Crit. Tables, Vol. 5, p. 235 et seq.

lost by the drop is set equal to the heat of fusion multiplied by the mass of the drop.

$$H_e = .00309\Delta T \times t = 80\frac{\pi(.1)^3}{6}. \qquad (15)$$

Table VII shows for average values of ΔT, the lowest value of T reached by drop, distance traversed by drop below 0° C. level and temperature gradient, as calculated from equation (15). A gradient of .01° C./m. is just about 5° F./1000 ft. and is roughly the gradient recorded by Carroll and McAvoy when they obtained ice at 31° F., 1° F. below freezing.

TABLE VII.

Av. ΔT °C.	Min. T °C.	t sec.	d cm.	$\frac{T}{d}$ gradient.
.5	− 1	25.6	10240	.01° C./m.
1	− 2	12.8	5120	.04° C./m.
2	− 4	6.4	2560	.16° C./m.

It has been common opinion that ice formed upon aircraft only when the temperature was within one or two degrees centigrade of the freezing point of water. The above calculation shows that glaze ice, under a condition of temperature inversion, would be expected to accumulate in this particular temperature range.

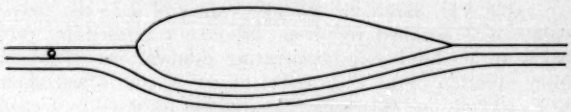
The records,[15] previously referred to, indicate that over the New York-Cleveland airway dangerous ice accumulations are encountered when the air temperature is as low as − 20° C. At these lower temperatures, the ice accretions must be due to the presence of fine water droplets in the fog, mist or cloud. The rapidity with which water droplets attain the temperature of the surrounding air, as shown by Table VI, implies that the fog, mist or cloud particles in the ice-forming region are undercooled. The fact that the depression of the freezing point of the water of the drops due to the curvature of their surfaces is at most only a few hundredths of a degree centigrade further confirms this conclusion.

In order to obtain an intimate picture of the way in which ice is formed on an airfoil due to droplets in the atmosphere,

[15] Loc. cit.

let the case of a strut with the flow lines be held in mind. A droplet is imagined to approach the strut coming down one of the tubes of flow. (See Fig. 9.) The effect of the velocity

FIG. 9.



Air flow tubes with droplet represented.

of fall of the droplets is neglected, for this ranges from .003 m./sec. to 8 m./sec. as compared with the flight speed of 50 m./sec. So far as the drops reaching the airfoil surface is concerned, it is not necessary to differentiate between the drops in the glaze storm and the droplets in fog, mist or cloud.

The forces which act upon the drop are the following: (1) A force arising from the pressure gradient normal to the lines of flow; (2) A viscous force due to the air which tends to drag the drop along with it. As the drop approaches the strut, force (1) tends to hold the drop away from the surface, where the curvature of the tube is convex toward the strut, and to force it on where the curvature is concave toward the strut; while force (2) tends to constrain it to a tube of flow and hence to carry it by without deposition. Force (1) is equal to the force that would hold a sphere of air of the same size as the drop in its path, regarding the air as non-viscous. This would be $(4/3)\pi a^3\rho_a(q^2/R)$ where ρ_a is the density of air and R is the radius of curvature of the tube at the point in question. This force is always normal to the tube and toward the center of curvature. Now the force required to hold the water droplet in the tube would be $(4/3)\pi a^3\rho_w(q^2/R)$ where ρ_w is the density of water, and is greater than the above in the ratio $\rho_w/\rho_a = 773$. Hence it is seen that the water droplet tends to cut directly across the tubes onto the strut. If force (2) is considered, however, this is not entirely true.

Force (2) is given by Stoke's law as $6\pi\eta av$, where η is the viscosity of air, a the radius of the drop and v the vector difference of velocity of the drop and neighboring air. The

acceleration due to it is

$$\frac{6\pi\eta a v}{\frac{4}{3}\pi a^3 \rho_w} = 4.5\frac{\eta v}{a^2}.$$

This shows that the smaller drops (when a is small) are accelerated from the stagnant point, S, much faster than the larger drops (when a is large). The result of the viscous force, therefore, is a separation of drops according to size, the smaller ones being carried farther from S. As Fig. 8 indicates, actual formations, under suitable conditions, show a solid hard cap symmetrically located over S with a scattering of crystalline rime at its borders which appears very much like little prisms standing up into the wind. Such deposits have been obtained many times at the leading edge of wings and struts upon planes in flight as well as upon the strut sections in the wind tunnel.

Taking an upper limit of v as 5000 cm./sec. and the value of η as 173.10^{-6}, the acceleration for fog particles ($a = 5.10^{-4}$ cm.) [16] equals 10^8 cm./sec.^{2} and that for moderate rain particles ($a = 5.10^{-2}$ cm.) [11] equals 10^4 cm./sec.^{2} The time required for a particle of air in a tube of flow to pass from stagnant point to point of reversal of curvature varies approximately from 5.10^{-5} to 3.10^{-3} sec., depending upon the size of the aerodynamic form. This gives for the maximum possible fractional change of velocity due to the viscous term for the fog particles from 1 to 60 and for the moderate rain particles from 10^{-4} to 10^{-2}. Hence it is seen that the viscous term accounts for the spread of particles according to size and for the observed change in character of the ice in the cap near the stagnant point on airfoils and struts. On wires this change could not be expected due to the small time allowed for the viscous force to act and the ice formation is due to a mixture of droplets.

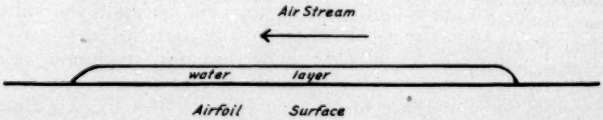
Ice formation on the airplane is due, in general, then, either to raindrops above the freezing point being deposited upon the airfoil surface in a condition of temperature inversion with the temperature only a degree or two C. below the

[16] Humphreys, "Physics of the Air."

freezing point or to fog, mist, or cloud particles at temperatures ranging from 0° C. to at least − 20° C. These latter particles are, as has been shown, undercooled and essentially at the same temperature as that of the air, in which they are found. In general, the size of the raindrop particles arising from the temperature inversion is considerably larger than the sizes of the droplets in fog, mist or cloud.

What is the history of these drops after they are deposited upon the airfoil surface? Consider the larger rain drops first. Due to their average large size, in all probability, there is little if any mass separation of these drops by the viscous force. Even if present, this scattering would not be observable for the drops are above the freezing point when they reach the airfoil surface and the water is free to move after deposition. Upon coming into contact with the surface, the inertia causes the drops to be spread out into a thin layer

FIG. 10.



Representation of the layer of water formed by a drop upon striking the airfoil surface.

(see Fig. 10). Instantaneously heat flows begin: (1) from the water layer into the airstream by the process of forced convection and (2) from the water layer into the airfoil surface by heat conduction. Freezing occurs, the rapidity of which depends upon the relationship of water temperature to air or airfoil temperature. If the conductivity of the surface is high and the temperature difference as large as allowable under these conditions, freezing undoubtedly sets in quickly at the interface of the water and surface, the ice crystals growing out into the liquid from the surface, that is, up the temperature gradient. At the air-water interface freezing does not occur due to the motion. The heat abstracted at this boundary is conducted from the liquid layer and serves only to assist in maintaining the freezing at the surface of the airfoil. The liquid water, due to its surface tension, as well as to the slowness of forced convection with

a consequent low rate of freezing at the surface, may actually spring up in the form of a drop, even though anchored by freezing at its base and may, as it reaches out into higher tangential velocity, be parted from the freezing layer and driven back over the surface of the airfoil.

The velocity of the air at the airfoil surface is zero. As long as the layer or film of water is thin, the air forces tending to remove it are exceedingly small and it is only when the drop grows and reaches out into the region of higher velocity that the forces tending to remove it become appreciable.

Since under the condition of temperature inversion the ice forms when the air-water temperature difference is only one or two degrees C., forced convection takes place slowly and the drops being large are given sufficient time to spring up, get into the higher velocity and hence to be driven back along the airfoil surface. The result of this is the form for this type of ice shown in Fig. 7. The drops move rearward contributing to the ice as they go. Further back on the airfoil they may be gathered into the air stream. Sometimes they cling to the surface, as they are driven back, forming huge icicles or even layers of ice over the surfaces of the airfoil. These icicles are frequently observed extending rearward from the trailing edge. The tendency for icicles to form is greatly enhanced when the temperature of formation is close to the freezing point.

Now consider the fog, mist or cloud particles, which are finer than the rain drops and undercooled. Due to their average smallness, mass separation takes place, distributing the particles as was seen in Fig. 8. When these particles strike the airfoil surface, they too are spread out into a thin layer by the inertia force as shown in Fig. 10. The droplets are undercooled and the surface may be supposed to supply crystallization nuclei, or initiation. Freezing occurs instantly at the water-surface interface due to the undercooling in addition to the factors considered in connection with the raindrops. The temperature difference between the water and the air or airfoil is considerably greater and the heat flows are, in addition, more rapid. The drops are smaller and they are locked in the very positions where they strike the airfoil surface. Hence the ice form is a measure of the way in which the particles reach the airfoil.

It is clear that if the temperature is close to the freezing point, the degree of undercooling is small, the temperature differences are small and conditions more nearly approximate to those under which the raindrops are deposited. At lower temperatures, the degree of undercooling becomes sufficient to account for the freezing of an appreciable fraction of the drop upon contact with the surface, which seems to provide the initiation for the crystallization. For instance if the air temperature is − 10° C. and if the liquid droplet is at this same temperature, then upon coming into contact with the airfoil surface, approximately 10/80ths of the drop may freeze instantly without any assistance from conductivity or forced convection. Conductivity serves to withdraw the remainder of the heat required to freeze the entire drop and lower its temperature to that of the air. Forced convection dissipates this heat into the air stream.

It is to be expected then that there are not two distinct types of ice—one due to raindrops and the other to fog, mist or cloud particles—but a gradual transition from the raindrop form of ice to the typical crescent form of the under-cooled droplets.

Ice has been accumulated by airplanes at temperatures as low as − 20° C. This proves that there are undercooled water particles in the air at this low temperature. What the lowest temperature is, I believe, has not been determined. However, at the lower temperatures, the number density of the droplets is less and the ice forms more slowly. It would be interesting to know what the lowest temperature of undercooling in the atmosphere is, how dense the particles are and under what conditions these particles become ice spicules and hence harmless to the flight of aircraft.

After the liquid layer becomes frozen upon the airfoil, there are definite forces [17] which act upon the ice. (1) The natural adhesion of the ice for the surface upon which it is frozen is positive and characteristic of the surface. (2) The ice layer is pressed upon the surface by the pressure of the

[17] A device designed to measure adhesion quantitatively yielded results which were not altogether satisfactory. They are not therefore being included in this report. The data obtained indicated that force (1) is 3 or 4 times force (2) for ice to rubber and that force (2) is some 70 per cent. of that as calculated for a vacuum beneath the ice.

atmosphere. (3) Aerodynamic forces due to the flight speed of the airplane are superimposed upon (2) and tend to hold the ice formed upon the leading edge of the airfoil. The maximum positive aerodynamic pressure, termed the dynamic pressure, for an airfoil with a relative wind of 5000 cm./sec. is approximately equal to 1.5 per cent. of the atmospheric pressure.

The forces arising from the flight speed or the relative wind are of such a character as to hold the ice formed at the leading edge of an airfoil in stable equilibrium. Under ice forming conditions in the wind tunnel, when the crescent shaped strip of ice was loosened and removed to a point in advance of the airfoil, then released, it was observed to be replaced by the wind and to become refrozen to the airfoil surface. When the strip of ice was freed from the airfoil surface and slipped laterally through a considerable angle, it was observed to be returned to its equilibrium position where subsequent freezing fastened it.

Suggested solutions of the ice problem seem naturally to come under the following headings, namely, chemical, thermal and mechanical. Some aspects of the solution of the problem from these different standpoints will now be presented in the light of a large number of wind tunnel tests, actual flight tests and calculations based upon these tests as well as other data obtained for this study.

There has been a general opinion that the deposition of ice upon aircraft might be eliminated by treating the exposed surfaces with waxes, greases, oils or in general with water resistants. The underlying idea is twofold: (1) a droplet of water upon striking such a treated surface, after being spread out into a thin layer by the inertia, would quickly regain its approximately spherical form (the angle of contact being assumed greater than 90° to such a surface) and be blown away by the air stream; (2) such a water droplet, even though frozen before recovering its form and being forced off, might still be blown off if its natural adhesion to the surface were zero. Let us consider these possibilities in the light of the theory herein presented of the formation of ice from the water droplets.

Case 1 should offer a solution provided (1) the freezing

of the thin water layer takes place slowly, (2) a sufficiently high tangential velocity is present to eliminate the drop, (3) a water resistant surface is available at the freezing point of water. It is not clear that these requirements can be fulfilled. Although the airfoil surface consists in a thin heat insulating layer, still the thermal conductance is fairly high relative to such a thin layer of water. In the case of undercooled droplets, freezing is very rapid even though the conductivity of the surface is actually zero. The layer of molecules next to the surface is frozen and then the positive adhesive tension water to ice holds the drop. Other drops then proceed to build from it. Also the ice growth begins at the stagnant line where the tangential velocity is essentially zero and only irregularities of flow are available to remove the water layers. Further the water layer is very thin and resides essentially in the air stream boundary layer where the velocity is considerably below that in the free air stream. The surface tension and viscosity of water and probably its other properties are continuous at the freezing point and there is no reason to suppose therefore that water resistant surfaces do not exist at, and below, the freezing point.

Case 2 brings into consideration the forces which tend to hold the ice upon the airfoil surface. These have already been stated as due to (1) the natural adhesion, (2) the air pressure and (3) the superimposed aerodynamic pressure. If the airfoil surface possessed zero adhesion to ice, force (1) would be eliminated but forces (2) and (3) would remain. When once a drop is forced into a thin layer and frozen at the stagnant line, others may build the ice back from it because of the positive adhesion tension water to ice. Even when the drop strikes at a point removed from the stagnant line, the air pressure forces it against the surface and holds it there against lateral movement because of any microscopic irregularities of the surface to which the liquid water conformed upon striking. Neither of these two suggestions in themselves, therefore, seems to offer a solution.

Experiments have been carried out along these lines, however, and results are given more fully in another paper.[18]

[18] N. A. C. A. Technical Note No. 345, "The Prevention of the Ice Hazard on Airplanes," Geer and Scott.

It is sufficient to say that no surface which was water resistant at room temperature indicated the slightest effect in warding off the freezing droplets under ice forming conditions. This result may have been because the water resistant property vanished at the lower temperature or, as the author thinks more probable, because of the heat conductance together with the undercooling of the liquid drops. These factors evidently outweigh the predominance of the surface tension over the adhesion tensions which tends to return the liquid to its spherical form. Also, no surface treated with oils or varnishes impregnated with waxes or other lubricants indicated any tendency to prevent the ice formation or even to retard it but rather accelerated the initial rate of deposition.

These experiments with the use of water resistants and low adhesives are entirely in accord with the theoretical considerations herein presented and serve to confirm them.

In the above mentioned experiments, the use of oil for the production of a low adhesion surface was shown to be successful. The requirements for such a surface are: (1) An oil which is sufficiently permanent and non-drying and whose viscosity is not too great at ice forming temperature. Its freezing point should be well below the ice forming region. (2) A means of holding the oil in place for otherwise it is driven back and finally off the airfoil by the scrubbing action of the wind and rain. These requirements have been met by a specially compounded oil held by rubber as a vehicle.

Tests were conducted on the reduction of adhesion of ice to surfaces coated with varnishes impregnated with waxes or greases. These tests revealed that a thin heat insulating layer of rubber beneath the varnishes reduced the adhesion, at least in the wind tunnel tests, to an extent greater than that of any of the varnishes alone. This result leads one to believe that the firmness of the adhesion of the ice increases with the rapidity of the freezing of the liquid water and therefore should increase, under the conditions of these tests, with the lowering of the air stream and water droplet temperatures. Qualitatively this seemed to be confirmed.

The application of water soluble substances to the exposed parts of the airplane has been proposed for the prevention of the formation of ice. The principle involved is that the

water droplets upon meeting the surface form with the soluble materials a solution whose freezing point is below zero. The success of this method depends upon obtaining a solution in the liquid layer of sufficiently high concentration of the solute to reduce the freezing point below the normal ice forming temperature range. It should be, and is, entirely possible to accomplish this in the case of the glaze storm or temperature inversion, for the ice formation occurs at a temperature close to zero and the concentration of the solution required is not great. No soluble substances have been tested at the time of this writing which prevent ice formation at temperatures below $-4°$ C.[19] This may be due to the failure of the solute to dissolve sufficiently quickly in the case of the undercooled drops or it may be that the average concentration in the liquid layer is too low to prevent freezing at the lower temperatures. As there are practical considerations in the way of this method, no careful examination of the facts has been made.

Since the temperature range of ice formation is at least 20° C., the amount of solute required to lower sufficiently the freezing point of the liquid layer is excessive. Rain at a temperature above the freezing point of water is effective in dissolving the solute, and if encountered would leave the airplane surfaces unprotected for any ice conditions that might subsequently be met. A solute must be held by a vehicle or medium of some character such as a varnish or paint. When the solute has been dissolved, the base must be laboriously removed before the surfaces may be prepared for the next flight.

Undoubtedly the reason why the general trend of attempts to prevent ice formation on airplanes has been along the line of lubricants, that is, waxes, greases, and oils lies in the simplicity of such a solution. The airplane need be changed in no way and no special constructions are necessary. Simply the lubricant would be applied to the desired surfaces whenever the likelihood of ice formation presented itself. On the other hand, while the employment of heat for ice prevention is obvious, it does involve special constructions and may add

[19] N. A. C. A. Technical Note No. 339 (1930), "Refrigerated Wind Tunnel Tests on Surface Coatings for Preventing Ice Formation," Knight, and Clay.

some weight to the airplane which must be carried at all times. If, however, heat is to be used for this purpose, it is necessary to know at the start whether or not the heat dissipated from the engine is sufficient. This problem is the subject of part III of this article.

A mechanical means of eliminating ice after its formation upon certain of the exposed airplane surfaces has been devised as the result of tests conducted in the small scale wind tunnel. The device may be termed a pneumatic leading edge and consists simply in rubber sheeting which covers the leading edge of the aerodynamic form and extends back toward the trailing edge. Beneath the rubber sheeting there are located pneumatic compartments which upon inflation so change the grain of the rubber as well as the form of the surface that the ice fails to follow the changes, thereby becoming loosened and free to be removed by the airstream.

Wind tunnel experiments of the following types led to the full conception of this pneumatic leading edge:

Type 1. A strip of rubber sheeting was applied to the leading edge of an airfoil. It was cemented upon a mahogany or an aluminum model or upon a thin sheet of rubber backed by fabric which had previously been applied to an entire aluminum model. A small rubber tube introduced at one end of the rubber strip served for inflation. The airfoil was then subjected to ice forming conditions and the behavior of the device noted. Although several tests of this character were made, none indicated that such an arrangement would successfully remove the ice.

Type 2. Rubber bands were applied to the airfoils and arranged to be extended periodically either parallel to the leading edge or toward the trailing edge of the model. These arrangements acted upon the ice in a manner similar to that observed with type 1. In no case was the ice observed to be removed.

Rubber bands varying in width from 1/16″ to 1/2″, mounted transversely to the airstream, and periodically stretched along their length, failed to prevent the ice from forming upon them and, in the case of the narrow bands, from completely encircling them. The ice formed in spite of vigorous vibration induced by the airstream. This agrees

with the observation made in flight tests of the ice building around the thin flexible cable connecting the upper and lower ailerons in spite of the vibration of this flexible connecting member.

Type 3. An aluminum airfoil was covered with a thin layer of rubber sheeting backed by fabric and the rubber was impregnated with a low adhesion oil. A resistance wire carrying an electric current was held at the leading edge and against the airfoil surface, thereby serving to cut the ice as it formed. Upon test the wire effectively cut the ice cap into two halves, but the ice was held tightly to the low adhesion surface with no indication of being removed. Two resistance wires were mounted farther apart and served to cut the ice along lines where the tangential air speed was greater. Tests showed that the ice was still held upon the airfoil surface. This result seems definitely to prove that the atmospheric pressure, irrespective of the low natural adhesion, is able to hold the ice upon the surface in opposition to the forces of the airstream and of vibration.

Type 4. This was a combination of types 1 and 3. An aluminum airfoil was covered with a thin sheet of rubber backed by fabric. A strip of rubber sheeting was cemented upon this to form a pneumatic compartment at the leading edge. Two resistance wires were mounted, as described above. They served not only to cut the ice but to hold the rubber sheeting against the airfoil surface so that there were essentially two penumatic compartments, one on either side of the pair of wires. Two small rubber tubes, one leading to each of the air chambers, served for inflation. Under ice forming conditions, the rocking action of the rubber sheeting, due to periodic inflation, freed the ice which had already been cut into two halves by the resistance wires. The first successful elimination of ice, forming upon an airfoil under conditions simulating those in practice, was accomplished by the periodic operation of this device.

During the wind tunnel test of the model, the ice was allowed to attain a thickness of approximately 1/16 inch before inflation. The maximum movement of the rubber surface then required to remove the ice was approximately 1/16 inch. The feasibility of this method as applied to a

full scale airfoil is indicated by this small movement, for while the full scale airfoil dimensions are enlarged by a given factor the rate of ice formation remains nearly the same. It follows that the amount of movement required of the rubber surface for a full scale airfoil should be the same as for the model, and the malformation produced by the movement would be correspondingly small. In practice, the rubber surface would conform closely to the airfoil form and would not interfere with the air flow. When the ice attained a thickness of 1/16″ to 1/8″, a transient expansion of about the same amount would then accomplish its removal. Since the malformation of the surface upon inflation is of an order of magnitude comparable with that of the ice that is being eliminated and since it is only transient, the drag of the airfoil should not be greatly increased, and the flying qualities of an airplane so equipped should not be seriously affected.

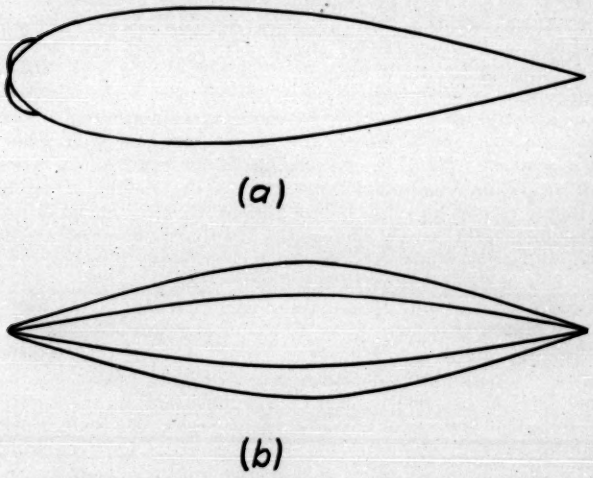
The next step was to attempt to constrain the rubber surface layer so that upon inflation the ice would be mechanically broken at or near the leading edge and hence be removed without the necessity of heating wires. This may be accomplished by producing a hinging or wedging action at the middle line of the ice forms. The methods adopted have consisted in the use of two pneumatic compartments parallel to the leading edge or of one large compartment extending well back on the airfoil or even to the trailing edge as in the case of a stream line wire. Diagrams of these ideas are shown in Fig. 11. The method may serve to protect extended surfaces, such as fuselage coverings and airship fabrics, by a suitable arrangement of pneumatic compartments. In order to reduce any increase in drag upon inflation of the device, the pneumatic compartments may be arranged to lie longitudinally.

For a description of the practical application of these experiments to the aeronautic ice-removing overshoes, the reader is referred to reference 13.

In none of the model wind tunnel tests has it been necessary to reduce the adhesion of the ice to the rubber surface by the application of oil to the rubber layer. The adhesion of the ice to the heat insulating rubber layer was, in all cases, sufficiently low to allow of successful operation of the devices.

In conclusion, it should be pointed out that while this mechanical means is applicable to wires, struts, radio masts, wing sections and the like, it is not easily adaptable to the

FIG. 11.



(a) represents a two compartment pneumatic leading edge as applied to a strut or wing section.
(b) represents the treatment of a streamline wire. There is only one air chamber.

propeller. As deposition upon this member occurs, vibrations are induced in the engine. The prevention of this remains an important problem.

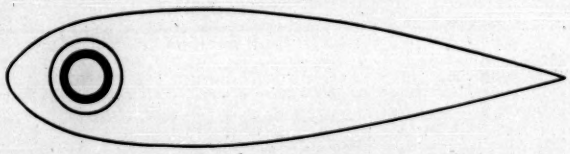
PART III.

Aircraft accumulate ice upon their exposed parts when operated under certain meteorological conditions as discussed in part II. These ice accretions may within a few minutes produce such vibrations in an airplane, and in time cause such malformation of its lifting and control surfaces, as to make flight hazardous. If means were available for maintaining the more important exposed surfaces of the airplane at a temperature above the freezing point of water, these accumulations of ice could be prevented and the usefulness

of the airplane as a commercial vehicle would be extended. The purpose of the study herein reported is to examine into the amount of power required to maintain the aerodynamic surfaces of the airplane at or above the freezing point of water under ice forming conditions.

As the miniature wind tunnel and models which had been used in the earlier studies of ice formation were available, they have been employed in these experiments. The data have been obtained from three different strut sections: (1) an aluminum U. S. Navy No. 1, (2) an aluminum R. A. F. 4Z, (3) a hollow copper R. A. F. 4Z. The first two models were drilled parallel to the leading edge so that electric heating elements [20] could be mounted within them. The third model had a heating element similarly mounted. (See Fig. 12.)

FIG. 12.



The electric heating element is shown mounted in a hole drilled parallel to the leading edge of the model strut.

These strut sections were in turn mounted in the tunnel, numbers (1) and (2) vertically and number (3) horizontally. Ice conditions were simulated. Observations were made at various temperatures of the power required to prevent the formation of ice under these conditions. The value of the power chosen for any given temperature was determined by decreasing the power until ice began to form, then by increasing the power until the ice melted and left the airfoil. The latter value seemed to be the same with the mist as without it.

[20] The ideal method of supplying heat to the surface of the airfoil for the purpose at hand is by condensation of vapor, for this method automatically supplies heat at the colder places, hence tending to maintain a uniform surface temperature in spite of differences in the rates of heat exchange at different parts of the airfoil due to differences in velocity, type of flow, etc. This method was tried but with such small models was impracticable and had to be abandoned.

Table VIII shows values for the power required to prevent ice for these three strut sections. P_1 is the power expressed in cal./cm. ° C. sec. required to keep the entire airfoil free of ice; P_2 is the power required to prevent ice at the leading edge.

TABLE VIII.

U. S. Navy No. 1 Aluminum Model.

Circumference 6.9 cm., airstream velocity 23.25 m./sec.

Temperature ° C.	Power$_1$ cal. / cm. ° C. sec.
− 4	.034
− 7	.037
−10	.043
Average	.038

4Z Aluminum Model.

Circumference 9.0 cm., airstream velocity 23.25 m./sec.

Temperature ° C.	Power$_1$ cal. / cm. ° C. sec.	Power$_2$ cal. / cm. ° C. sec.
− 9	.045	.042
− 7	.047	.037
− 5	.051	.036
− 3	.056	.036
− 1.5	.058	.035
Average	.051	.037

4Z Hollow Copper Model (Mounted Horizontally).

Circumference 9.0 cm., airstream velocity 23.25 m./sec.

Temperature ° C.	Power$_1$ cal. / cm. ° C. sec.	Power$_2$ cal. / cm. ° C. sec.
− 8.5	.053	.043
− 7.0	.046	.041
− 5.0	.050	.041
− 3.0	.053	.034
− 1.5	.053	.036
Average	.051	.039

In connection with the theory of ice formation as discussed in part II the following observations are interesting. The surfaces of the three airfoils are each water resistant, that is, are not easily wet by water. As long as the power of the heating element is as great as that recorded in Table VIII, the drops impinging upon the airfoil near the leading edge flatten into thin layers and then are observed to spring up and race

rapidly toward the trailing edge of the strut, very much like the movement of drops in the spheroidal state. As the power becomes less than the values given, the movement decreases in magnitude and intensity, then ceases. Freezing begins. The ice form which follows depends upon the proximity of the power to the values in the table. When far removed from these values, the ice is typical of the ice that would form at the airstream temperature, but when the power is just below the value required to prevent ice the character of the formation resembles more nearly glaze ice.

It is now necessary to establish a method of obtaining full scale values from those given in Table VIII. The one employed is based upon dimensional reasoning and is taken essentially from Gibson.[21] Let it be assumed that the only quantities involved in the process of heat exchange are the following:

P_e = the heat exchange per unit of time,
ρ = density of fluid,
l = a linear dimension of the surface,
v = velocity of fluid relative to surface,
σ = specific heat of fluid,
θ = temperature difference,
k = conductivity of fluid,
μ = viscosity of fluid,
ν = kinematic viscosity of fluid.

The following four independent and non-dimensional constants may be formed

$$\frac{P_e}{\rho l^2 v^3}, \quad \frac{\sigma\theta}{v^2}, \quad \frac{k}{\sigma\mu}, \quad \frac{\nu}{lv}.$$

An expression for P_e may then be written

$$P_e = c_1 \rho l^2 v^3 \varphi_1 \left(\frac{\sigma\theta}{v^2}, \frac{k}{\sigma\mu}, \frac{\nu}{lv} \right), \tag{16}$$

where c_1 is a non-dimensional constant. Since for gases under ordinary conditions $k/\sigma\mu$ is essentially constant this

[21] Gibson, *Phil. Mag.*, **47**, 1924, p. 324.

factor may be omitted. Experiments by Gibson indicate that $P_c = c_2\theta^m$ where c_2 and m are constants. Then

$$P_c = c_3\rho l^2 v^3 \frac{\sigma^m\theta^m}{v^{2m}}\varphi_2\left(\frac{\nu}{lv}\right),$$

where c_3 is a new constant. The experiments of Hughes,[22] Gibson and others show that

$$P_c = c_4 v^n, \tag{17}$$

where c_4 and n are constants. Incorporating this, the equation becomes

$$P_c = c\rho l^2 v^n(\sigma\theta)^m\left(\frac{\nu}{l}\right)^{-(n-3+2m)}.$$

The heat exchange per unit length per degree difference in temperature is

$$\frac{P_c}{l\theta} = c\frac{\rho\sigma^m\theta^{m-1}v^n d^{n-2+2m}}{\nu^{n-3+2m}}, \tag{18}$$

where d is any linear dimension of the body, that is, the heat exchange per unit length per degree difference in temperature is proportional to d^{n-2+2m} for a given velocity and proportional to d^{2m-2} for a given value of vd.

Equation (18) is the desired relation for obtaining full scale values from those given by the model experiments. The exponents in the equation must be experimentally determined for any given surface. For certain cases they are given by Gibson [23] and Hughes.[22] For the calculations at hand it seems best to choose $n = .62$ as given by Hughes, and $m = 1.02$ as derived by Gibson.

There has been general opinion that the heat dissipated by the aviation engine is insufficient to maintain the necessary exposed surfaces of the airplane at or above the freezing point of water at ice forming temperatures. As this is contrary to the conclusion of this study, it is desirable as far as possible to check the results of the model tests with data obtained in similar experiments by other experimenters. For this

[22] Hughes, *Phil. Mag.*, 31, 1916, p. 118.

[23] Gibson, loc. cit.

purpose the data given in Table VIII are now compared with data on heat exchange obtained by Hughes for a symmetrical airfoil section of circumference equal to 23.1 cms. Hughes' data were taken for a temperature range between room temperature (not stated in report) and steam at atmospheric pressure. By employing his data,[24] assuming room temperature to be 20° C., and making use of equation (4) for converting to same velocity, the power dissipated by forced convection for the symmetrical airfoil is

$$P_c = \frac{1682}{600 \cdot 80}\left(\frac{23.25}{12.172}\right)^{.62} = .0524,$$

$$P_c = \frac{1594}{600 \cdot 80}\left(\frac{23.25}{10.784}\right)^{.62} = .0535,$$

$$P_c = \frac{1190}{600 \cdot 80}\left(\frac{23.25}{6.652}\right)^{.62} = \underline{.0538},$$

$$\text{Average } .0532 \frac{\text{cal.}}{\text{cm.} \cdot ^\circ \text{C. sec.}}.$$

Again employing equation (4) for converting the above value of the power to correspond to the same linear dimension as the R. A. F. 4Z airfoil

$$P_c = .0532\left(\frac{9.9}{23.1}\right)^{.66} = .0304 \frac{\text{cal.}}{\text{cm.} \cdot ^\circ \text{C. sec.}}.$$

This may now be compared with the values given in Table VIII. In all cases the latter values are larger. This should be expected from a comparison of the methods of obtaining the data. Hughes measured the power by weighing the water collected from the condensation of steam in the airfoil. This implies an essentially perfect means of supplying heat to the surface for this purpose, for the condensation will occur only when the temperature of the surface tends to drop below the boiling point and in this way maintains a uniform temperature of the airfoil surface despite any variation in the heat exchange at different parts of the surface due to differences in velocity, type of flow, etc. On the other hand, with the electric heating element the distribution of heat is not such as to

[24] Hughes, loc. cit., Table IV, case 2, p. 128.

maintain a uniform temperature of the airfoil surface. With airstream velocity established, the temperature of the leading edge was in all cases higher than that of the trailing edge. The power required to prevent ice from the entire airfoil and measured in this way should be greater therefore than the power that would be necessary if the temperature of the airfoil surface were everywhere the same. Also the power so determined should be expected to be larger than that obtained as a measure of heat exchange for the observer tends to choose a value of the current too high rather than too low for the prevention of ice, that is, a safe upper limit. Considering these differences in the methods of obtaining the values, the check between the two sets of data is satisfactory.

Gibson's data on heat exchange for two circular pipes are now compared with the airfoil data given by Hughes. The rates of heat exchange for the pipe of diameter 9.52 cms., the airfoil whose equivalent diameter is 7.35 cms. and the pipe of diameter .925 cms. are respectively .042, .033, and .01 cal. per degree per cm. per sec. Scaling the latter two values to that of the large pipe, we find

$$P_c = .010\left(\frac{9.52}{.952}\right)^{.66} = .0469,$$

$$P_e = .033\left(\frac{9.52}{7.35}\right)^{.66} = .0417,$$

to be compared with

$$P_e = .042.$$

The close agreement of these values indicates that the airfoil data fit admirably into those for a pipe of the same circumference. Now it has been shown theoretically by Reynolds,[25] Rice,[26] and others that the heat exchange and the air resistance should go hand in hand. Rice collects experimental results which prove this in the case of pipes both below and above the critical velocity. The resistance of the airfoil form used by Hughes must be less than that of a pipe of its equivalent

[25] Reynolds, *Proc. Manchester Lit. and Phil. Soc.*, 1874. Reprinted in *The Steam Engine*, Perry, p. 594. Also reprinted in "Heat Transmission by Radiation, Conduction and Convection," Royds.

[26] Rice, "Industrial and Engineering Chemistry," May 1924, p. 460.

circumference. Since Gibson's data were based upon a long series of experiments, which showed remarkable check by use of equation (18), not only among themselves but by comparison with data by King,[27] it follows that the value of the power given by Hughes must be too large rather than too small, and that the data of Table I are safe upper limits to the power required to prevent formation of ice.

The question now arises whether these data for the model airfoils will provide reliable full scale values when converted by equation (18). Gibson shows by comparison of his data with that of King that equation (18) is entirely satisfactory for a ratio of diameters as large as 3170, the largest diameter being 9.52 cms. and the smallest .003 cms. Since the ratio of the linear dimensions of the R. A. F. 4Z model to full scale is only approximately 40, it appears to be safe to base conclusions upon these data in spite of the fact that the full scale dimensions are larger than any for which equation (18) has been tested. This conclusion is further supported by the fact that the resistance of aerodynamic forms is known to be principally viscous and the flow to be laminar.

From Table VIII a value of the power exchange equal to .04 cal. per cm. per degree C. per sec. is chosen and this is converted to correspond to a velocity of 50 meters per second, and to a linear dimension of 375 cms. These values represent full scale. By use of equation (4)

$$P_c = .04\left(\frac{50}{23.25}\right)^{.62} \cdot \left(\frac{375}{9.9}\right)^{.66} = .706 \frac{\text{cal.}}{\text{cm. } ^\circ \text{ C. sec.}}.$$

The power exchange for an extended surface as calculated from data taken from the International Critical Tables is 3.12 cal. per cm. per degree C. per sec., a value 4.42 times as large as the above. However it seems impossible to compare these values since in the latter calculation the aspect ratio [28] (the ratio of the surface dimension transverse to the airflow

[27] King, *Phil. Trans.*, A, CCXIV (1914), p. 373.

[28] The Int. Crit. Tables give a summary only of the data on forced convection. By reference to Rice's original article (ref. 18) a general equation for heat exchange is given which takes account of the size factor. This equation has not been used in the above calculation because sufficient data for its application are not available.

to the dimension longitudinal to the airflow) is not taken into account and this obviously has a direct bearing on the result. Clearly the higher the aspect ratio the greater the rate of heat exchange should be. This point is discussed later.

As a practical example, assume a 20° C. range for ice formation, a monoplane, whose span is 41 ft., wing area 275 ft.^{2} (average span 6.7 ft.), and a speed of 180 miles per hour.

$$P_e = .04\left(\frac{80.3}{23.25}\right)^{.62}\left(\frac{422}{9.9}\right)^{.66} = 1.03\,\frac{\text{cal.}}{\text{cm.}\,^\circ\,\text{C. sec.}},$$

$$\text{H.P.} = \frac{1.03 \cdot 41 \cdot 30.48 \cdot 20 \cdot 4.18}{746} = 144.$$

It requires then 144 H.P. to maintain such a wing at a temperature 20° C. higher than that of the surrounding air. When ice begins to form, additional heat must be supplied to the surface upon which deposition occurs equal to the product of the heat capacity of the water reaching the airfoil surface into the temperature difference between the water and zero degrees C. For the rates of deposit met in practice, this power is negligible in comparison with the above, as is shown by calculation using observed rates of accumulation as well as by the model experiments in which the power required to prevent ice formation was practically the same with, as without, the mist.

An airplane of the dimensions given would probably mount an engine of 450 H.P. An aviation engine turning at cruising speed dissipates into the jacket (or the fins) and into the exhaust an amount of power which is approximately the same and equal to the torque horse power. The present data indicate then that under ordinary conditions of operation the exhaust or the jacket heat is sufficient for the prevention of ice formation upon the wing.

There appear to be two possible methods of employing the heat dissipated from the engine for the prevention of ice. The exhaust gases may be passed through suitable ducts metallically connected to or integral with the leading edges of the surfaces to be protected. Or the exhaust gases may serve as a source of heat for a vapor system. These two methods are briefly discussed in the following paragraphs.

Because of the corrosive action of gases and vapors upon metals, pipes or ducts to confine them are desirable. Calculations based upon data[29] for free convection have been made of the power dissipated from pipes (radiators) located within an airfoil and maintained at an average temperature of 270° C. or 100° C. The former temperature corresponds approximately to the case of exhaust gases and the latter to that of vapor at atmospheric pressure. In order that the power absorbed by the pipes and dissipated therefrom may equal that exchanged by forced convection at the surface of the airfoil, the area of the pipes must be unreasonably large. As this implies weight increase, which is always undesirable, an alternative must be sought. The one suggested consists in ducts for the gases or vapor integral with the metallic surface of the leading edge. Fig. 13 is a diagram of such an arrangement and indicates its thermal effectiveness.

It is evident that the heat is conducted to the immediate surface as fast as it is dissipated. This heat follows back over the surface tending to warm it. To this are added the heat conducted through the surface sheeting and that carried to the sheeting by free convection within the airfoil. Due to the air velocity a large part of the heat absorbed by the duct from the gases or vapor is effective in warming the surface of the airfoil.

The back pressure produced upon the engine in the case of using the exhaust gases may be relieved by discharging the gases into the air at a point on the airfoil where the pressure is below atmospheric.

In the second method a jacket surrounds the exhaust pipe and contains a non-freezing liquid of high heat of vaporization and of low density. The vapor is conducted through pipes to the ducts in the wings and tail surfaces. After condensation the liquid is either allowed to flow back to the boiler or is pumped back. The method demands baffles in the ducts to prevent changes of load distribution upon banking or other maneuvers of the airplane. While this method is simple indeed and allows of heating the cabin and supplying hot water at the same time, it has the disadvantage of the added weight of the jacket and the liquid which must

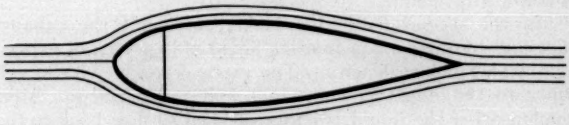
[29] International Critical Tables, loc. cit.

be carried. On the other hand, the corrosion induced by the vapor might be better controlled than with the hot exhaust gases.

Calculations based upon data given by Herreshoff [30] for the application of steam cooling [31] to automobiles indicate the feasibility of the vapor system for the solution of the ice problem.

In order to examine the variation of the heat exchange along the surface of the airfoil, an experiment was devised in which the heat dissipated from narrow strips [32] located on the airfoil surface was measured. The airfoil with the resistance strips is diagrammed in Fig. 14. When the airfoil is mounted in the wind tunnel and subjected to wind of a given velocity with a given air temperature, the heat dissipation is measured for a given surface temperature of the strip, as measured by a thermo-junction. The three strips employed are so located on the surface of the airfoil as to be representative of the three parts of the airflow about the airfoil, (1) near the leading edge, (2) at the point of maximum ordinate, (3) near the trailing edge. It was hoped to learn whether the heat dissipations were directly proportional to the air velocity or dependent further upon the vortices of the flow; also to what extent the heat dissipated from some strip upstream raises the temperature of a strip located downstream with respect to it, and hence to obtain an idea of the thermal effectiveness of the arrangement of Fig. 13.

FIG. 13.



The diagram shows the duct provided at the leading edge of the wing section for the exhaust gases or vapors.

As a result of a series of tests the following values for the heat loss from the strips were determined. (See Table IX.)

[30] Herreshoff, *J. of S. A. E.*, Feb. 1926.

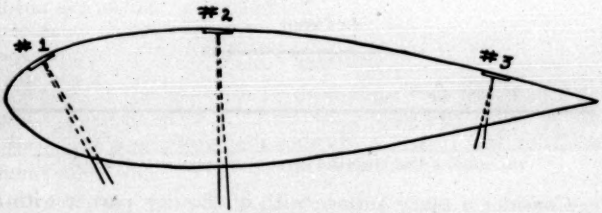
[31] Steam or vapor cooling of the airplane engine would permit the use of the jacket heat as the source of vapor. This method appears favorable in that the parasitic resistance of the cooling system of the engine might be eliminated.

[32] A similar method was used by Jakeman. See Royds (Ref. 18), p. 121.

TABLE IX.

Angle of Airfoil.	Heat dissipated in $\frac{\text{cal.}}{\text{cm.}^2\,^\circ\text{C. sec.}}$		
	Strip No. 1.	Strip No. 2.	Strip No. 3.
0°.....	.0198	.0218	.0231 average .0215
10°.....	.0202	.0281	.0103
20°.....	.0232	.0257	.0086

FIG. 14.



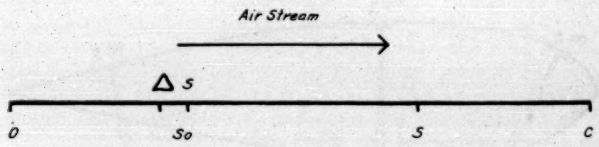
Resistance strips are shown mounted in the surface of the strut for studying the heat exchange at different points.

As the powers given in Table IX are not proportional to the velocities as determined by measuring the pressures at the three strips for the different positions, it appears that the heat dissipation is not proportional to the velocity. However, due to the smallness of the model and the inability to measure accurately small changes in the quantities concerned, this question remains unanswered.

The rate of heat exchange, averaged for the entire airfoil and based upon the value .04 cal. per cm. per degree per sec. taken from Table I, is .00404 cal. per cm.^{2} per degree per sec. This is less than one-fifth the value for the strips as given in Table II. The reason for this is that the heat dissipated into the air by the parts of the airfoil upstream warms the parts of the airfoil downstream. If the velocity of airflow is very high the boundary heat film remains thin downstream. It is conceivable in this case that a large fraction of the heat dissipated into the film upstream might be absorbed by the surface downstream because of the impartation of the

heat motion of the molecules to the metallic airfoil surface. The effect of aspect ratio is therefore evident in that the longer the longitudinal dimension of the airfoil is, the larger the amount of power absorbed from the air. The strips have a high aspect ratio (assumed infinite) whereas the R. A. F. 4Z airfoil has a finite aspect ratio. The value of the power exchange averaged for the entire airfoil should therefore be expected to be less than that for the strips. The following discussion is intended to clarify this point.

FIG. 15.



The surface of chord c is divided into strips ΔS normal to the airstream.

Consider a plane surface with air flowing past it with a given velocity. (See Fig. 15.) Let the surface be divided into strips of width ΔS normal to the airflow. Suppose heat to be supplied from below to a unit strip, ϵ, located at S_0, and to maintain its temperature above that of the main airstream by an amount ΔT_0. Assume the elemental strip, ϵ, raises the temperature of all points downstream according to

$$\Delta T = \Delta T_0 e^{-a(S-S_0)}, \tag{19}$$

where ΔT is the difference in temperature between a point S and the main body of the airstream. The total difference in temperature of S due to all the strips upstream with respect to it is

$$\Delta T_T = \sum \Delta T \cdot \frac{\Delta S}{\epsilon} = \sum_0^S \frac{\Delta T_0}{\epsilon} e^{-aS} \Delta S,$$

$$\Delta T_T = \int_0^S \frac{\Delta T_0}{\epsilon} e^{-aS} dS,$$

$$\Delta T_T = \frac{\Delta T_0}{\epsilon a} [1 - e^{-aS}].$$

Next assume that sufficient heat is automatically supplied to

each element to bring its temperature difference to ΔT_0. The total power required is

$$P_e = \int_0^c kl\left[\Delta T_0 - \frac{\Delta T_0}{\epsilon a}(1 - e^{-as})\right]dS,$$

where k is the power per cm.^{2} per degree and l the length of the surface transverse to the airstream.

$$P_e = kl\left[\Delta T_0 - \frac{\Delta T_0}{\epsilon a}\right]c + \frac{kl\Delta T_0}{\epsilon a^2}[1 - e^{-ac}] = \bar{k}l\Delta T_0 c, \quad (20)$$

where $\bar{k}$ is a value for the exchange coefficient averaged for the entire airfoil. Its value is

$$\bar{k} = k - \frac{k}{\epsilon a} + \frac{k}{\epsilon a^2 c}[1 - e^{-ac}]. \quad (21)$$

The error in the last term by dropping e^{-ac} is less than 1 per cent. provided $ac > 5$. And since $c = 4.95$ cms. in our experiment and since data indicate $a \geq 1.0$, this condition is fulfilled. Approximately and numerically then

$$\bar{k} = k - \frac{k}{a} + \frac{k}{a^2 c}. \quad (22)$$

Corresponding numerical values of a and $\bar{k}/k$ are given in Table X.

TABLE X.

a.	$\bar{k}/k$.
.8	.066
1.0	.202
1.2	.308
1.4	.389

Although the development given above applies to a plane surface with uniform airstream velocity, data obtained from the airfoil experiments should give approximate check. From the data for prevention of ice, the power dissipated was given as .04 cal. per cm. per degree per sec. or .00404 cal. per cm.^{2} per degree per sec. The average value of the power dissipated as given by the strips is .0215 cal. per cm.^{2} per degree per sec. These are respectively $\bar{k}$ and k. It follows therefore that

$$\bar{k}/k = \frac{.00404}{.0215} = .188,^{33}$$

[33] This could be checked by a modified form of equation (18) if the experimental values of the exponents were known.

and from the table it is seen that a should be equal approximately to 1.0.

By observing the rise in temperature of strips 2 and 3 due to a given amount of power in strip 1 and the rise in temperature of strip 3 due to a given amount of power in strip 2, the value of a can be calculated by use of the assumed relation

$$\Delta T = \Delta T_0 e^{-a(S-S_0)}.$$

The average value obtained is 1.1 and serves to check the above theory.

The interesting result is the effectiveness of this method of heating provided the heated surface has a low aspect ratio, that is, a large chord relative to its length transverse to the airstream. For a chord of length only 4.95 cms. and a velocity of 23 meters per sec. as shown above, the average k may be taken as only .2 that for a strip of very high (infinite) aspect ratio in calculating the heat exchange by

$$P_e = \bar{k} l \Delta T_0 C.$$

The conclusion of this study is that the heat dissipated by the airplane engine is sufficient for the prevention of ice formation upon the important exposed parts of a monoplane. The heat of the exhaust gases or of vapor, if the latter method is used, may be absorbed by ducts integral with the leading edges of the wing and the tail surfaces and distributed by them in an effective manner to the surfaces of these parts. At least in the cases of using the exhaust gases the propeller may also be protected from ice formation. This may be accomplished by conducting the hot gases to the interior of a hollow metallic propeller by a sliding valve and allowing it to discharge from exit ports near the propeller tips. Although no practical tests of the above outlined methods of preventing ice formation have been made, the practicability and the simplicity of these means seem available when occasion necessitates.

THE OPTICAL SENSITIZING OF SILVER HALIDES BY COLLOIDAL SILVER.*

BY

S. E. SHEPPARD.

The first observation of the optical sensitizing of the silver halides was made by Ed. Becquerel,[1] in his discovery that silver chloride papers, after exposure to blue-violet or white light gave enhanced (visible) image formation when further illuminated with yellow or green light. He termed this "the continuing action of light." It was shown by Lüppo-Cramer in 1909 [2] that this is due to direct optical sensitizing by colloid silver. Lüppo-Cramer synthesized silver photo-halides in gelatin, by combination of colloidal silver with colloidal silver halide † followed by coagulation with acid and after washing, peptization in gelatin by excess halide ion. These synthetic photo-halides are sensitive to light beyond the blue-violet, and indeed give good panchromatic sensitizing—up to the infra-red.‡

These observations have been confirmed by other observers.[3] They make it probable that the panchromatic sensitizing of ordinary gelatino-bromide plates when treated with bisulfite followed by washing in alkaline water, discovered by J. G. Capstaff and E. R. Bullock,[4] is also due to sensitizing with colloidal silver.

Experiments by Wightman, Trivelli and Sheppard [3] with colloidal silver sulfide in place of colloidal silver have shown that this can also form a photo-halide, which is optically sensitized.

The fact that this type of optical sensitizing of the silver halides is of little practical value compared with that procured

* Communication No. 442 from the Kodak Research Laboratories.

† This method is possible for bromide and iodide, not for chloride. *Cf.* Lüppo-Cramer, *loc. cit.*

‡ It is an interesting question as to whether the remarkable infra red sensitive emulsions obtained by Abney (See *Eder's Hdbuch. der Phot.;* Vol. II, Halle, 1898 p. 382; also H. Lehmann, *Arch. wiss. Phot.*, II, p. 216) were connected with colloid silver sensitizing.

by the use of dyes has resulted in relatively little attention being paid to it. In particular, there has been little attempt to explain the mechanism of this optical sensitizing. Lüppo-Cramer appears to identify it with the analogous action of organic sensitizing dyes. This may be the case, but, since the mechanism of optical sensitizing by these is also in question, does not carry us much farther.

Arising out of their work on the optical effects on silver halides of adsorbed ions and molecules [5] Fajans and his collaborators have indicated applications to the action of colloidal silver. The investigations of Fajans and Frankenburger showed in particular that silver bromide having silver ions adsorbed ("silver body") had its threshold of visible decomposition in light displaced from about 460 mμ to beyond $\lambda > 615$ mμ "or perhaps even into the infra-red." * Later investigations of Fromherz [6] indicated that silver ions adsorbed to silver bromide in gelatino-silver bromide emulsion extended the absorption from 500 to 700 Å.U. toward the red. (It must be noted that this was associated with a definite increase of absorption in the whole spectral region studied, hence involved not only wave-length displacement, but increase of total number of atoms active at a given wave-length.) He did not succeed in measuring spectral sensitizing (for development) with adsorbed silver, owing to fog formation but did for thallous ions, which also produced an extension—about half as great as silver ion—of the spectral absorption.

The optical effects of adsorbed ions with silver halides are considered by Fajans from two viewpoints. In the case of adsorbed anions—e.g., OH^-—a sensitizing effect is predicated on the ground of their relatively lower electron affinity than in the case of the halide ion. At the same time, the deforming effect of the cation (Ag^+) on these adsorbed anions comes into play. In the case of adsorbed cations—Ag^+ and Tl^+—an original explanation [7] in terms of lower electrostatic work-function at the surface was found incompatible with the behavior of bromide ions. Fajans,[8] therefore, attributes their action to deformation of the (surface) halide ions—"such a bromide ion surrounded on all sides by silver ions [in the lattice] will be specially strongly deformed on the part of the

* It is perhaps questionable whether the material was free from colloid silver.

adsorbed silver ions, since the field of *this* silver ion is not screened by neighboring bromide ions." Similarly for thallous ions.

Fajans remarks that finally "in the third type of optical sensitizing by occluded nuclei of elemental silver it is a question of the displacement of spectral sensitivity into the long wave region. Eggert and Noddack [9] have shown that this is accompanied by a parallel displacement in the same sense of the absorption of the *silver halide* * in the case of silver chloride emulsion. Whether the alteration of the electron orbits of the halogen ions which thus appears is specifically produced because the occlusion of the metal nuclei causes a gap in the normal lattice [10] or whether the neutral metal atoms exert forces on the ions of the adjacent lattice, may be left for consideration. In any case such a displacement of the absorption into the long wave region is often caused by lattice disturbances, and influences other related phenomena, such as photoconductivity."

It appears from this that Fajans considers the optical sensitizing effect of colloidal silver to be caused by an altered absorption of light by deformed silver halide. We will put aside for the present the question as to whether ionic deformation, in Fajans' sense, *necessarily* involves a shift toward the long wave-lengths. That question is discussed in another communication dealing with the light absorption of the silver halides.[11] The question at present is, whether the optical sensitizing by colloidal silver is due to changed absorption of the silver halide *per se*, or whether it is the absorption of light by the colloidal silver itself which is responsible. In the latter case, the phenomenon is indeed analogous to optical sensitizing by dyes. With these, the sensitizing is undoubtedly conditioned by the absorption spectrum of the dye, modified, it is true, by its adsorption to the silver halide, and therefore, very possibly, by deformation of the large complex molecule.[12] Fajans cites in connection with this interpretation the work of Eggert and Noddack on the quantum yield of silver halide in green light after preillumination,[13] almost the only quantitative work so far on the Becquerel effect. This important investigation does not, however, entirely sup-

* Italicized by the writer.

port Fajans' thesis. Eggert and Noddack found indeed that, calculating from the silver yield in green light (546 mμ) on the basis of quantum equivalence (I Ag atom per $h\nu$), only a small part of the total absorption by the preformed silver could be active in the subsequent green illumination. An active absorption of 0.18 per cent., as follows for quantum equivalence with green light, corresponds to only 20 to 30 per cent. of the total absorption by the silver present of this light. Hence, they conclude, only a small part of the silver present is photolytically active.

They remark that the cause of the optical sensitizing is a very delicate question. Fundamentally it must be attributed to the silver formed by preillumination, but "the immediate cause is none the less to be sought elsewhere."

Their interpretation is that the new green absorption is produced "when under certain conditions, the adsorbed silver and the adsorbing atom or molecule mutually deform each other with production of a previously non-existent absorption band." To this absorption region they ascribe the primary energy consumption. In the sense of this conception they regard at least one molecule of AgCl [per how much silver?] as concerned in the absorption, and as decomposed thereby.

Granting this, their next conclusion does not appear to be necessary or adequate. "According to this conception, the process is no longer to be regarded as a sensitization. Rather do we regard the so-called green sensitization of pre-exposed silver chloride as a normal photolysis of an adsorption-combination arising through pre-exposure between silver chloride and silver." It is not easy to see that this helps one to penetrate into the mechanism of the effect. If the "normal photolysis" of silver halide be, as present evidence [14] strongly indicates, the process

$$X^- + h\nu \rightarrow X + \theta,$$
$$Ag^+ + \theta \rightarrow Ag_{met.},$$

then for their conception we must write

$$\{Ag_n\}\, Ag^+\, X^{-*} + h\nu' \rightarrow \{Ag_n\}\, Ag^+ + X + \theta$$
$$\{Ag_n\}\, Ag^+ + \theta \rightarrow Ag_n + 1,$$

where Ag_n stands for a *limited* amount of colloidal silver, having Ag^+ ions adsorbed, and X^{-*} for the thereby deformed halide ion. In this sense, the primary absorption is due to the halide ion, in agreement with Fajan's conception. It is a principal point of Eggert and Noddack's argument that the absorption of photochloride is due to the *combination* of silver halide and part of the colloidal silver. They observe that on fixing the plates the rose to purple color disappeared, leaving only the "brown color of colloidal silver." This argument does not appear very weighty, however. Colloidal silver sols of rose to purple color, i.e., of absorption similar to the photohalides, can be readily prepared in absence of silver halide, and are converted, by electrolytic coagulation, to brown-black silver, e.g., by treatment with fixing salts. There is no proof, therefore, that the silver halide was necessary. In addition, retarded development processes with silver chloride emulsions are well known which yield rose red (and other colored) colloidal silver images which withstand the action of fixing salts. Without in the least reflecting upon the value of Eggert and Noddack's investigations, it does not appear that they have reached finality on the problem. In particular, their emphasis on the specific maximum absorption—and sensitivity—in the green (550–560 mμ) does not appear to exhaust the possibilities of optical sensitizing by colloidal silver —to which must be added, through later researches,[3] colloidal silver sulfide. In Capstaff and Bullock's experiments, sensitizing developed as a progressive extension to the red, which passed into a uniform panchromatizing effect extending to the infra-red. Again, while a form of silver *sulfide* sensitizing by bathing silver bromide plates with allyl thiourea, followed by alkali, gave relatively high green and yellow sensitivity, and very slight red sensitivity,[3] direct "photo-halide formation" with colloidal silver sulfide gave high yellow and red sensitivity, and very little sensitivity in the green.[3] It appears evident that both colloidal silver and colloidal silver sulfide *can* effect optical sensitizing each in quite diverse spectral regions, or over quite wide regions. The ascription of this to differential degrees of deformation of the halide ion in the photohalide, rather than to the specific absorption, related to size and structure, of the colloidal metal or metalloid sulfide, would

be a rather artificial and *ad hoc* extension of the deformation hypothesis.

The mechanism of this effect is therefore still very much an open question. If, receding from the ionic deformation conception of Fajans, and Eggert and Noddack, we revert to the analogy with dye sensitizing suggested by Lüppo-Cramer and Eder, we find that however strong a case may be made for this analogy, it does not help materially in penetrating the mechanism of the action. Not only is the exact mechanism of optical sensitizing by dyes itself still obscure [15] but the comparison is difficult. On the one hand, in the case of dyes, we have certain specific complex molecular structures, which are themselves responsible for the selective light absorption. These absorptions are associated with mobile electrons, but have not yet been completely interpreted in terms of the modern theory of molecular spectra. The theories proposed are:

(*a*) Formation of excited molecules of dye—already adsorbed to surface of silver halide—and transfer of absorbed energy to silver halide (or halide ion) by second order collisions, similarly to the photochemical sensitization of gas reactions by excited mercury atoms.[16] We should have

$$D + h\nu \rightarrow D^*,$$
$$D^* + \mathrm{Br}^- \rightarrow D + \mathrm{Br} + \theta,$$
$$\theta + \mathrm{Ag}^+ \rightarrow \mathrm{Ag}_{\mathrm{met.}}$$

This would permit the same molecule of dye to facilitate the decomposition of a theoretically unlimited number of silver halide molecules, since the dye molecule emerges unchanged. There is no direct evidence on this, but some evidence that the chemical instability of the dyes in light is increased by silver salts.[15] In the case of small amounts of dyes adsorbed to emulsified silver halides, this hypothesis is very difficult to test experimentally. (*D* can also *accept* Br.)

(*b*) The adsorbed dye molecules, or ions, may give up electrons on absorption of light in their own absorption band.[15] These electrons then reduce silver ions similarly to those discharged from halide ions by blue-violet light. The residual organic radicle then undergoes further chemical change. The investigation of Goldmann [16] on the photo-potentials produced by dyes at metal electrodes may be regarded as sup-

porting this view, and the photo-voltaic effect [17] promises to be a valuable method of studying the sensitizing process. In the meantime, we may notice that the two principal classes of sensitizing dyes, *viz.*, the *phthaleins* and the *cyanines* (polymethine) form *negative* and *positive* electrolytic ions in solution respectively. If their adsorption to the silver halides is an electro-adsorption, then they will be adsorbed to cations and anions respectively.[15]

$$\begin{array}{c|c|c} Na^+ & Phthalein^- + Ag^+ & Br^- \\ Cl^- & Cyanine^+ + Br^- & Ag^+ \end{array}$$

It is easier to conceive of the phthalein giving up its valence electron to the adsorbed silver ion, than of the positively charged cyanine (isocyanine, carbocyanine) ion losing an electron. Yet the optical sensitizing behavior of both classes of dyes is, so far as ascertained, very similar.*

(*c*) The dyes in question are mostly fugitive in light, i.e., are bleached or oxidized, and this change is accelerated by silver salts, both dissolved, and as solid absorbents. The sensitizing effect may consist in a photo-reduction by the dye, connected with the electronic transfer. A hypothesis put forward by Kögel and Steigmann [18] supposes that the dye in light acts as a reducing agent by way of hydrogen released from water.

It will be seen that theories of dye sensitizing range from a physical transmission of energy to rather complex chemical reactions. It is possible that investigations of the time element, e.g., by way of intermittent exposures, may lead to some decision here, but at present it is evident that the mechanism of dye sensitizing is no better known than that by colloidal metals.

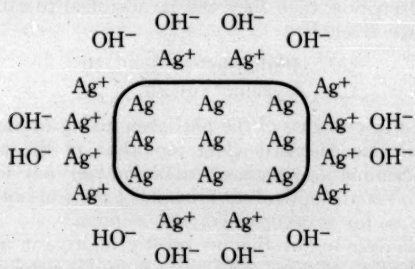
It is, therefore, even possible that this latter may be more readily understood, and may help elucidate the case of dyes. If we put aside the deformation conceptions of Fajans' as a *complete* explanation, we have as possible explanations the following mechanisms—at least.

(*a*) Resonance absorption of light [19] in colloid particles, involving definite excitation of certain loosely bound atoms or

* Comparative experiments on intermittency and reciprocity failure on silver halide sensitized with the two classes of dyes should give interesting information.

atom-groups, and transfer of energy by second order collisions. This would be strictly analogous to hypothesis (a) for organic dyes.

(*b*) The colloid particle may be regarded as stabilized by an ionic adsorption layer, whether of OH^- ions (or silver hydroxide) or of halide ions (or silver halide).



Discharge of electrons in this stabilizing field might lead to aggregation of particles, and formation of nuclei for development, as suggested in F. F. Renwick's theory of latent image formation.[20] The objection to this is that it actually begs the question, as to how the specific light absorption, related to particle size, of the colloid metal, can afford, the energy necessary for the anionic discharge—which would normally be at much shorter wave-lengths. This brings us back to the deformation suggestions, so that no actual advance is made.

(*c*) A third proposal which is now brought forward is that of sensitized photo-electric emission. As pointed out by the writer,[21] the red limit of the external photo-electric effect for silver metal is about 3300 Å.U. As this corresponds to the resonance (excitation) potential of metallic silver atoms, a simple monatomic layer could not be expected, from Ives and Olpin's results with the alkali metals[22] to give electron emission for longer wave-lengths. If this restriction were absolute, it would appear useless to appeal to the photo-electric effect. Considerable recent work on thin composite layers of metals and metal oxides, however, tends to show that very considerable displacement of this threshold is possible.[23] Campbell states that cæsium with potassium suitably de-

posited in composite film gave stable sensitivity to red light, and relatively strong currents at 8000 Å.U.

Similar results have been reported by other observers.

It could be suggested that these results may be applied, without consideration of their origin, to the photo-halides. It appears desirable, however, to indicate in what manner this sensitized photo-emission is produced in composite films, and why it may be feasible to consider a similar condition to be responsible for optical sensitizing of silver halides by colloidal metals.

EXTERNAL FIELDS AND ELECTRON EMISSION.

The emission of electrons from metals and oxides, either under the influence of light or of heat, is greatly influenced by an applied external electric field. In the case of heat (thermionic emission), the work function ϕ in the expression

$$I = AT^n e^{-\phi}/RT$$

for the thermionic current, is reduced so that eventually at sufficiently intense fields, electron emission from cold metal becomes possible. The thermionic work function ϕ is related to the long wave threshold of the photo-electric effect; either

$$h\nu_0 = \phi,$$

or [24]

$$h\nu_0 - p = \phi.$$

Considerable recent work supports the conclusion that the "true" value of ϕ for uncontaminated metal is $= h\nu_0$, i.e., the thermionic and photo-electric work-functions are identical.

Schottky's [25] theoretical investigation of the effect of external fields was made in terms of the electrical image-force at the surface. Beyond a distance of 10 to 20 atoms ($2 - 4 \times 10^{-7}$ cms.) from the surface of a conductor up to distances of about 10^{-4} cm., the force acting on an electron, even at high electron densities (saturation currents) is practically entirely due to the image force, the influence of the electron on the neighboring conductor. If x is the distance from the metal surface, then this image force is given by $F = -eV_i$ where $V_i = e/4x$. Generally, with an external field X

$$V = \frac{e}{4x} + Xx$$

where e = charge on electron, and the denominator 4 occurs because the distance from the optical image of the electron is double its distance from the metal surface. The potential V_0 at distance x_0 is obtained for $dV/dx = 0$, when $V_0 = (eX)^{1/2}$. V_0e is the decrease in the work function produced by the external field. Schottky assumes that all electrons striking the surface normally with kinetic energies greater than $\phi = V_0e$ will escape, while those with energies less than $\phi - V_0e$ will return into the metal. Schottky's expression has been found generally justified, but it is important to notice that the magnitude of the effect (as regards total number of electrons liberated *and* the V_0e term) under a given field has been found very much larger than the apparent external field warranted. In this connection Schottky has pointed out the great increase in local potential gradients which would be produced by microscopic and ultramicroscopic unevennesses of the metal surface. He estimated that the true field strength at the roughest parts of a surface might be greater by a power of 10 than that estimated for the smooth surface. This is borne out partly by experiments of the Research Staff of the General Electric Company, London;[26] L. Nordheim[27] has shown that on the wave-mechanics theory, electrons can still escape, albeit in much diminished numbers, at lower values of the work-function than given by Schottky's expression. The calculation of the total work of emission even in a relatively simple case is very complicated, but, as pointed out by R. H. Fowler,[28] does not entirely reconcile the orders of magnitude of the calculated and observed emissions in intense fields.

So far as concerns optical sensitizing of silver halides, the facts of chief importance are:

(*a*) The lowering of the thermionic work-function of metals by applied (external) electric fields.

(*b*) The low thermionic work-function and photoelectric thresholds of composite films, e.g., sodium on tungsten, still more of cæsium on oxygen on tungsten. Such composite films are partly to be regarded as a succession of monomolecular layers, but with other important factors intervening (see later).

(*c*) The increased effect of applied (external) electric fields on the work-functions and photoelectric thresholds of such composite films.

Thus E. O. Lawrence and L. B. Linford [29] have found recently that "the photoelectric thresholds of films of the alkali metals are shifted by large amounts toward the red by application of quite ordinary electric fields. Thus a field of 69,000 volts/cm. at a surface of potassium on oxygen on tungsten extends the photoelectric threshold in the infra-red * from 8880 Å. U. to 10,600 Å. U. This corresponds to a reduction of the work function of the surface of about 0.23 volts. A field of 49,000 volts/cm. removes the threshold of a pure potassium film on tungsten from 5600 Å. U. to 5900 Å. U. corresponding to a reduction of the work function by 0.12 volt." They point out as a noteworthy further fact that their metal data indicate that not only thresholds, but the whole spectral sensitivity of the surface so far as investigated is shifted to the red to the same extent as the threshold.

The suggestions which we wish to make are, *first*, that the increased sensitivity of multiple or composite films is due to autotonic (Volta) electric fields, of micro-elements produced in "forming" the films, and *second*, that similar conditions are induced in sensitizing silver halides with colloid metals.

CONTACTS, COHERERS AND VOLTA CONDENSERS.

In the important paper of Schottky [25] on electron transfers already referred to, he discusses conditions for conductance between metals held at different potentials (by an external applied e.m.f., or otherwise) in relation to the shape of the junction, and of the potential gradient at an edge.

The manner in which the small p.d. corresponding to Volta series difference of the metals could give rise to relatively (and absolutely) intense fields over short distances is still more clearly indicated by P. W. Bridgman [30] in a discussion of the emission of electrons in strong fields. He points out that with two different metals, in metallic contact at one part, and bent to oppose their surfaces at another "the Volta potential difference may be made to generate a field of any desired intensity." The distance between the plates at 10^{-6} cms., would give a field of 10^{6} volts/cm.—assuming a dielectric constant of 1 (vacuum). If we suppose a speck of metal exposed to this field, in the case of composite films, it does not

* Already beyond the resonance (1st series) line of potassium.

appear impossible that these minute systems would act like an external (applied) electric field to lower the photoelectric threshold, and the thermionic work-function of the specks.

Some support is given to this hypothesis by the investigations of J. A. Becker [31] on "Phenomena in Oxide Coated Filaments." He finds that from a comparison of these filaments, and filaments with composite surfaces such as thorium on tungsten, cæsium on tungsten, and cæsium on oxygen on tungsten, it appears probable that barium oxide coated (tungsten) filaments owe their high (thermionic) activity to adsorbed metallic barium. In "forming" the filament surface, evidence is produced for an electrolysis of barium oxide giving metallic barium. If oxygen is beneath the adsorbed barium, it increases the activity; if above, it decreases it. It appears likely that in the "well broken down" filaments described by Becker, at a number of points metal arcs will form across the oxide layer, as in experiments on the electrolysis of fused and solid salts. With barium on oxygen on tungsten, the probability of local Volta condensers giving intense local fields seems considerable.

It may well be that there is insufficient experimental warrant for assuming such Volta elements, and that their existence, if assumed, is inadequate to explain the facts. Becker and Mueller,[32] from their investigation and demonstration of intense electrical fields near metallic surfaces, formulate the following conclusion: "To sum up the picture to which we are led: The adsorbed thorium or other electropositive material *produces fields which, close to the surface, are very large* and *in such a direction to help electrons escape;* at larger distances the fields are in the opposite direction, and may be of appreciable magnitude as far out as 2500 thorium atom diameters." Later they say "Whatever the *explanation* for the fields in question may be . . . for the present we wish merely to point out that the surface fields obtained by this method are of such striking magnitude that they *require an explanation*, and that quite likely when we have found this explanation we shall know more about the mechanism of emission from composite surfaces."

In proposing the Volta couples as effective in this connection, we were attracted by its phenomenological simplicity,

which admits a quantitative formulation of the sensitizing effects, and facilitates to some extent its application to the optical sensitizing of the silver halides. Suppose V be the Volta p.d. between the coupled metals. Then we may write, to a first approximation,

$$h\Delta\nu = \frac{Ve}{d},$$

where $h\Delta\nu$ is the primary sensitizing effect, e = electron charge, d = distance. We can perhaps get some conception of the magnitudes involved from E. O. Lawrence and L. B. Linford's results.[29] They found a field of 69,000 volts/cm. at a surface of potassium on oxygen on tungsten extended the threshold in the infra-red from 8800 Å. U. to 10,600 Å. U.—a reduction of $h\nu_0$ at surface of about 0.23 volts. Now Ives and Olpin [22] found the extreme threshold for pure monatomic potassium at 7700 Å. U., near the resonance potential 7899 Å. U. of 1.60 volts. So the composite film, potassium on oxygen on tungsten was giving a threshold about 1000 Å. U. further into the infra-red, in Lawrence and Linford's case, i.e., a displacement equivalent to 0.18 volt. Assuming correspondence of the applied and "self" fields, we should have a Volta p.d. of $4.5 - 1.60 = 2.9$ volts to produce a field of 54,000 volts/cm. This would be effective at a separation of about 500 Å. U., or 5×10^{-6}/cm., which is not an improbable magnitude.

Transferring the conditions to sensitizing by colloidal silver, we have the problem of the primary Volta p.d. for one and the same metal, which has to provide a field of the order producing a shift in threshold of silver from 3300 Å. U. (3.70 volts) up to say 7300 Å. U., or about 2 volts. The voltage drop Ag/Ag^+, i.e., for silver metal in a silver salt solution is about 0.75 volts.[33] It may be assumed that as between relatively massive silver, and silver atom layers (colloid silver) adsorbed to silver halide, Volta p.ds. of the order 0.5 volt may exist. On the basis of Lawrence and Linford's results, assuming about 10^6 volts/cm. to give a displacement $h\Delta\nu$ of 2 volts, for a p.d. of 1 volt we should require separations of 10^{-6} cm., for 0.5 volt, 0.5×10^{-6} cm., or about 5×10^{-8} cm., i.e., 50 Å. U. This is not much more than 10 times

the interval of a silver metal lattice, but perhaps indicates that amicrons of colloidal silver in the field of ultramicrons of colloid silver could emit electrons, in light of wave-length down to 8000 Å. U. We are assuming that the colloid silver in question is adsorbed to the surface of silver halide. The electron thus emitted tends to pass the positive pole of the Volta couple. If it is taken up by an Ag^+ ion (of the silver halide) we have a redistribution of silver, but no *increase* of metallic silver, since each electron emitted leaves an Ag^+ ion as residue. In this respect the mechanism suggested accords with F. Weigert's conclusions on the photo-accomodation of photo-halides in polarized light.[34] The intimate relation of the Weigert effect to the Becquerel effect makes this concordance interesting. Since both the contact p.d. available, as well as the interval, are necessarily statistically distributed, the production of optical sensitizing over a wide region, i.e., panchromatic sensitizing, is also explained. Furthermore, since these effects change the local dispersity or distribution of colloid silver, the local fields would alter autocatalytically, as observed for the sensitizing. However, for colloid silver completely occluded in, and to some extent for colloid silver strongly adsorbed superficially to, silver halide, the conditions are modified, owing to the dielectric change.

R. Jaeger [35] gives the dielectric constant of AgBr as about 12. The effect of this on the assumed Volta element can be considered as correspondingly dividing (for the simplest geometrical conditions) the interval over which the same field is effective; this would bring the effective interval down to distances of the order 6 Å. U. for a field of 10^6 volta/cm. On the other hand, a particle of silver embodied in a dielectric such as silver halide, having a heteropolar lattice structure, may be regarded as thereby already subject to a high polarization owing to the fields of the ions, which might amount to an effect of "saturation," so that the assumed "external field" of the ultra-microscopic Volta condenser would involve no further effect. This would depend, however, upon the size and shape of the particle, differing for a fluid electrolyte with freely mobile ions. In the extreme case, however, we should apparently expect to find the energy required to remove an electron from such a metal particle greatly reduced. Joffé [36]

has pointed out that the effect of increased dielectric constant on sodium metal in NaCl is to reduce the energy required in the proportion of $1/K$ for single metal atoms. It would be-become less for particles of larger size, and indeed, probably in *proportion to their own Volta field* sketched here. Since single metal atoms changing to metal ions at one point, to give a metal atom at another point, would only very slightly alter the distribution, the possible extension into the infra-red, at wave-lengths of the order 10,000 to 30,000 Å. U. would hardly be appreciable. How such conditions might modify the internal photo-conductance effect with silver halides requires further examination. It appears that as the dispersed silver approached the surface, and as the particles responding to the Volta field of other particles approached a certain magnitude, the observed effects (induced optical anisotropy) might become more marked.

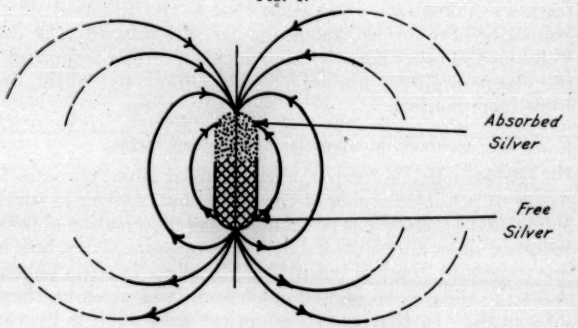
ADSORPTION, DEFORMATION AND ORIENTATION.

The assumed Volta p.d. with colloidal silver particles is supposed possible because of the adsorption of silver to silver halide, and a consequent polarization and deformation of both adsorbed silver and silver halide. The question arises, how is this suggested "optical sensitizing" by silver or silver sulfide related to the general photographic sensitizing given by these substances? In that case, the optical absorption is by the silver halide itself, and the presence of silver or silver sulfide nuclei raises the photographic sensitivity because of this, without changing the spectral distribution.[37] I have previously suggested [38] that in optical sensitizing by colloid silver or sulfide the sensitizer was very finely distributed over the grain, and perhaps also through the grain, in a very disperse form, therefore, as compared with a quite small number per grain of relatively large nuclei in the case of general sensitizing. Therefore, in optical sensitizing the sensitizing action was limited chiefly to the immediately contiguous silver halide, the halide ions of which were supposed greatly deformed, while in general sensitizing by large nuclei a greater extension of the field about the nucleus was supposed to exist, and hence a greater orientation effect.

This conception can be partly retained but modified,

according to the present hypothesis. Thus it is still necessary to suppose that optical sensitizing by colloid silver implies large numbers of finely divided silver particles—many small enough, in fact, to be affected by the fields due to larger ones. But the wave-length sensitizing is now regarded as due primarily to Ag atoms, not bromide (halide) ions; *in absence* of this amicroscopic phase, a few ultramicrons of silver or silver sulfide (or both together) act as nuclei. The Volta field of these nuclei, particularly if anisotropic, will extend around the Volta condenser, according to Fig. 1.

FIG. 1.



Field about colloid silver or silver sulfide nucleus.

Each interval or line through the AgBr dielectric connecting massive silver with adsorbed silver or sulfide is a field line, corresponding to the stress lines previously suggested about the nucleus.[38] An *orientation* of electron-wavelets when a quantum of energy is absorbed in this field seems entirely reasonable, since there is nothing otherwise to determine definitely by which atom the incident radiation should be absorbed, or, in other words, at what points of a wave-front a quantum will occur. Optical sensitizing affects the wave-length active, the energy-level; general sensitizing affects the (Heisenberg) indetermination of quantum (total energy) incidence. Or, as I have already phrased the position, on classical theory, sensitizing may be either in respect of *wave-length* or *phase*.[39]

RELATION TO PHOTOELECTROLYTIC HYPOTHESIS.

In a paper on the red-sensitizing action of soluble iodides and cyanides,[40] I suggested that this was due to an orienting action of these anions on the redistribution of colloid silver, related to the Weigert effect, and possibly to be interpreted as a photoelectrolytic effect, in view of Tubandt's recent work on unipolar conduction in solid electrolytes. Since then A. P. H. Trivelli [41] has developed a specific photoelectrolytic theory of silver sulfide sensitizing and latent image formation. The Volta elements described in the previous section are obviously very much the same as those presupposed in Trivelli's theory, and, therefore, the question arises wherein the present hypothesis differs from his. The difference is the stress laid on electrolysis. In Trivelli's theory, the silver bromide is a mixed conductor; the primary freeing of electrons by light is of importance as in some way setting up an active e.m.f. by the nucleus, with regard to which the silver halide now behaves as an electrolyte, with migration of Ag^+ ions to the cathodic Ag-pole, and discharge there as metallic silver.

On the other hand, in the hypothesis of Volta condensation sketched here, it is the dielectric strength of the silver halide which is emphasized, and thereby the possibility of field strengths of great intensity over *short* intervals, capable of affecting the photoelectric emission of electrons from silver (or sulfide) amicrons in the field (Becquerel effect) *or*, over wider intervals, affecting the incidence of quantum absorption by bromide ions and electronic displacement (orientation). Although this conception does effectively bring in line the earlier hypothesis of deformation and orientation with Trivelli's suggestion of photo-voltaic elements, it is apparently and really in antagonism with the suggestion of photoelectrolysis. The antagonism is evident, in that electrolytic conduction would represent a decrease of the dielectric strength, and a reduction of the external field. Since an actual electrolysis would mean in any case deposition of silver on part of the nucleus, if it occur, it might only imply a reduction in range of the—herein assumed—primary sensitizing. It is hoped to obtain experimental evidence on some of the factors involved, in conjunction with Mr. Trivelli and Dr. Vanselow.*

* The actino-dielectric effect in CaS-Me phosphors studied by P. Lenard and his school is very suggestive in this connection.

Finally, it should be pointed out that the hypothesis of the Becquerel effect (colloid silver optical sensitizing) is not necessarily restricted to the formation of Volta elements. The existence of intense fields at short distances from composite films has been demonstrated by Becker and Mueller, and they may be due entirely to electrostatic forces at interfaces not definitely involving formation of such elements. In that case the conception of deformation and orientation remains in the form already suggested.[20] But even if artificial, the application by Bridgman of the Volta condenser, and its relation to Trivelli's photocell, seem well calculated to the analysis of the phenomena. As already stated, the adsorption of silver atoms, and their deformation by silver bromide [20] is here regarded as the primary condition permitting Volta difference in the colloid silver.

Quantitatively, the proposed hypothesis for the Becquerel effect involves only *redistribution* of silver, the processes being

$$Ag + h\nu' \rightarrow Ag^{+} + \theta.$$

Amicrons in surface field are formed and

$$\theta + Ag^{+} \rightarrow Ag_{met.}$$

built on to ultra-microns. Although this is in accord with Weigert's investigation of the accommodation phenomena of the photo-halides in polarized light, it does not, any more than Weigert's results, agree with Eggert and Noddack's [9] result for the visible Becquerel effect with photo-chloride produced by illuminating a Valenta emulsion. This is a silver chloride emulsion with free *silver* or soluble silver salt, as silver citrate, tartrate or nitrate, in the ratio of 30 per cent. to 70 per cent. Ag salt. There must, from the *increase* of silver which they found, have occurred an equivalent decomposition of either the AgCl, or the Ag salt. Eggert and Noddack consider this to have been the AgCl, but do not offer proof that it was this, and not the soluble silver salt which was decomposed.[42] This point is secondary, however, to that of the steady production of *new* silver in the green light after preillumination with blue light. If the hypothesis of the "outer field" effect or Volta condensation is to be maintained, it must be considered that

this acts not only on the Ag particles in the field, but also on the halide or other negative ions, causing them to discharge for lower quantum values; that is, the processes

$$Ag + h\nu \rightarrow Ag^{+} + \theta$$

and

$$X + h\nu \rightarrow \quad X + \theta$$

must be coupled, and the deforming effect of adsorption on the X^{-} ion (e.g., Cl^{-}) regarded as supported by the action of the field. The extent to which such coupling occurs is, however, very uncertain. In contrast to the results of Eggert and Noddack, F. Weigert [43] in his remarkable work on photodichroism and photoanisotropy has given repeated evidence that in the "accommodation process" (the sensitized process) no *increase* of silver occurs.

It is perhaps significant that Weigert finds the intensity of the accommodation effect to *increase* with longer wave-length. This is what we should expect if the coupling of the "normal" and the "sensitized" actions was progressively reduced with wave-length increase—assuming diminishing activation of X^{-} ions, compared to colloid Ag action.

The antagonism (synergy) of the Becquerel effect with the normal latent and visible image formation is of great interest in relation both to the Weigert effect and the Herschel effect, and will be considered more fully in these connections later.[44]

SUMMARY.

1. Theories on the optical sensitizing of silver halides are discussed.

2. It is suggested that optical sensitizing by colloid silver (Becquerel effect) is primarily due to sensitized photoelectric emission by silver amicrons and ultramicrons.

3. Sensitized photoelectric emission of composite metal films is considered as due to minute Volta condenser fields.

4. Self-fields as origin of sensitized photoelectric effects are considered in relation to the Becquerel and Weigert effects with silver halides.

ROCHESTER, N. Y.,
January 20, 1930.

ADDENDUM.

After this article had been completed, there appeared in the *Physical Review*, Jan. 15, 1930 (Vol. 35, p. 158) a paper by Neil B. Reynolds on the Schottky Effect and Contact Potential Measurements on Thoriated Tungsten Filament, in which the contact potential or Volta *p.d.* between "patches" of the two metals of the filament is used to account for the deviation from Schottky's relation at low electric fields. It is stated that "the low electronic work function of a composite surface such as thorium on tungsten has generally been attributed to the effect of the contact difference of potential existing between the thorium and tungsten atoms at their point of contact," and reference is made to O. W. Richardson's papers (*Phil. Mag.*, **43**; 557,1922).

The theory of "patches" of one metal (thorium) on the other (tungsten) which they develop and which accords with their results is not necessarily inconsistent with the hypothesis suggested in the foregoing article, which supposes the field of the two conjoined metals to affect the threshold of a metal particle in an adjacent dielectric. It is evident that the photoelectric and photographic phenomena can only be compared within certain limits, since the means of detection and observation are so radically different.

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[2] Lüppo-Cramer, *Phot. Korr.*, **46**, 269, 339, 579, 1909; **47**, 21, 1910.

[3] Cf. E. P. Wightmann, A. P. H. Trivelli and S. E. Sheppard, JOUR. FRANK. INST., **204**, 491, 1927.

[4] J. G. Capstaff and E. R. Bullock, *Brit. J. Phot.*, 67, 719, 1920. Cf. also A. P. H. Trivelli and S. E. Sheppard, "Silver Bromide Grain of Photographic Emulsions," Monograph No. 1 from the Kodak Research Laboratories, pp. 53, 54, 55.

[5] Cf. K. Fajans, "Eder's Handbuch der Photographie," Halle (Saale), W. Knapp, Ed. 3, **2** (1), 633, 1927.

[6] H. Fromherz, *Z. physik. Chem.*, **1B**, 324, 1928; H. Fromherz and G. Karagunis, **1B**, 346, 1928.

[7] K. Fajans and W. Frankenburger, *Z. physik. Chem.*, **105**, 255, 1923.

[8] K. Fajans, op. cit.

[9] J. Eggert and W. Noddack, *Z. Physik*, **31**, 922, 1925.

[10] Cf. S. E. Sheppard and W. Vanselow, *J. Phys. Chem.*, **33**, 250, 1929.

[11] To be published as a continuation of op. cit. (10).

[12] Cf. S. E. Sheppard, *Chem. Rev.*, **4**, 319, 1927.

[13] J. Eggert and W. Noddack, op. cit. (9).

[14] Cf. S. E. Sheppard: Sixth Hurter and Driffield Memorial Lecture, *Phot. J.*, **68**, 397, 1928.

[15] Cf. (12), also S. E. Sheppard and H. Crouch, *J. Phys. Chem.*, **32**, 751, 1928.

[16] A. Goldmann, *Ann. Physik*, **27**, 449, 1908.

[17] W. Vanselow and S. E. Sheppard, *J. Phys. Chem.*, **33**, 331, 1929; S. E. Sheppard, W. Vanselow and V. C. Hall, ibid., p. 1403.

[18] G. Kögel and A. Steigmann, *Z. wiss. Phot.* **24**, 18, 1926.

[19] Cf. F. Kirchner and R. Zsigmondy, *Ann. Physik*, **15**, 573, 1904. But cf. also G. Mie, *Ann. Physik*, **25**, 429, 1908.

[20] F. F. Renwick, Hurter Memorial Lecture, *J. Soc. Chem. Ind.*, **39**, 156, 1920; cf. also S. E. Sheppard, *Brit. J. Phot.*, **68**, 5, 1921.

[21] S. E. Sheppard, *Phot. J.*, **70** (n.s. **54**), 132, 1930.

[22] H. E. Ives and A. R. Olpin, *Phys. Rev.*, **34**, 117, 1929.

[23] N. R. Campbell, *Phil. Mag.*, **8**, 667, 1929.

[24] Cf. E. H. Hall, *Proc. Nat. Acad. Sci.*, **15**, 504, 1929.

[25] W. Schottky, *Z. Physik*, **14**, 63, 1923; *Phys. Z.*, **15**, 872, 1914.

[26] N. R. Campbell, *Phil. Mag.*, **6**, 633, 1928; *ibid.*, **8**, 667, 1929.

[27] L. Nordheim, *Z. Physik*, **46**, 833, 1928; *Proc. Roy. Soc.*, **121A**, 626, 1928. R. H. Fowler, and L. Nordheim, *Proc. Roy. Soc.*, **119A**, 173, 1928.

[28] R. H. Fowler, *Proc. Roy. Soc.*, **122A**, 36, 1929.

[29] E. O. Lawrence and L. B. Linford, *Phys. Rev.*, **34**, 1492, 1929.

[30] P. W. Bridgman, *Phys. Rev.*, **34**, 1411, 1929.

[31] J. A. Becker, *Phys. Rev.*, **34**, 1323, 1929.

[32] J. A. Becker and D. W. Mueller, *Phys. Rev.*, **31**, 438, 1928.

[33] Cf. A. J. Allmand, "Principles of Applied Electrochemistry," New York, Longmans Green & Co., 1924.

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[36] A. F. Joffé, "Physics of Crystals," New York, McGraw Hill Book Co., Inc., 1928.

[37] S. E. Sheppard, Colloid Symposium Monograph, **3**, 75, 1925.

[38] S. E. Sheppard, *Phot. J.*, **68** (n.s. **52**), 412, 1928.

[39] S. E. Sheppard and A. P. H. Trivelli, op. cit. (4), p. 50. S. E. Sheppard, *Brit. J. Phot.*, **68**, 4, 1921.

[40] S. E. Sheppard, *Phot. J.*, **62** (n.s. **46**), 88, 1922.

[41] A. P. H. Trivelli, *Phot. J.*, **68** (n.s. **52**), 14, 67, 1928.

[42] Cf. F. Weigert and W. Schöller, *Sitzsb. preuss. Akad. Wiss.*, **39**, 641, 1921, and discussion by Lüppo-Cramer in "Grundlagen der phot. Negativerf.," in Eder's Handbuch der Photographie, Ed. 3, vol. 2, part 1, p. 39 et seq.

[43] Cf. particularly F. Weigert, *Z. physik. Chem.*, **3B**, 377, 1929.

[44] As bearing on this, there may be cited the following papers of F. Weigert and collaborators, which were not received before completion of the present article: *Z. physik. Chem.*, **3B**, 389, 1929; **4B**, 83, 239, 258, 1929.

Note on Two Apparent Large Temporary Local Magnetic Disturbances Possibly Connected with Earthquakes. S. CHAPMAN. (*Terr. Mag. and Atmospheric Elec.*, June, 1930.) The two magnetic occurrences to be described here were brought to the attention of the author by the president of the Section of Seismology of the International Union for Geodesy and Geophysics. While a steamer, the *West Holbrook*, was entering the Gulf of Tokio at 6:40 P.M., Aug. 3, 1926, a slight tremor was felt. "From the above position to Yokohama the ship's compasses were acting queerly, having on northerly courses a deviation of 0°.5 west; where before there had been 3° west. Upon arrival at Yokohama learned that an earthquake had taken place. The following morning, August 4, azimuths were taken for recording deviation. The compasses were still found to be out. On August 5 azimuths were again taken and compasses were now found to be back to normal." This return to normalcy makes it probable that the effect on the compasses was not due to some mechanical result caused by the shock.

The second magnetic change was noticed on a ship 4° or 5° from the epicenter of the Corinth earthquake of April 22, 1928. At 18:20 G.M.T. it was seen that the two independent gyro compasses varied 5° in their indication of direction from the two magnetic compasses. The first pair remained in agreement all the time. All difference between the two kinds of compasses disappeared shortly after 19:10 G.M.T. No electrical changes on the ship could account for the divergence. The earthquake did not come until about two hours after the magnetic disturbance. It may be that the causes of the earthquake in some way produced the magnetic effect. There was no magnetic storm nor any electrical activity in the atmosphere that could account for the phenomenon. There is no continuous magnetic record kept at any place near the site of the occurrence. The record from the nearest observatory in Helwan showed nothing unusual. In Great Britain there are four observatories keeping continuous records but earthquakes do not occur near them. Thus owing to the rarity of observatories in other parts of the earth such brief magnetic variations might easily fail to be recorded, and, in the absence of gyro compasses, the change might pass unnoticed on the vessel itself.

G. F. S.

A CHART OF CONSECUTIVE SETS OF ELECTRONIC ORBITS WITHIN ATOMS OF CHEMICAL ELEMENTS.

BY

VLADIMIR KARAPETOFF,

Cornell University, Member of the Institute.

According to the generally accepted theory of atomic structure, much of the behavior of the atom of a chemical element can be accounted for by assuming it to consist of a positively charged central nucleus (which contains practically the whole mass of the atom) and of a number of negatively charged electrons revolving around the nucleus in approximately elliptic orbits, in a manner somewhat similar to that of the planets rotating about the sun. Since these orbits have never been actually observed, their physical existence can be neither insisted on nor denied. More scientifically stated, an atom behaves *as if* it possessed such revolving electrons. When electronic orbits are discussed below, this limitation of "as if" is always understood. In some respects such orbits simply denote energy levels. When it comes to the simplest diatomic molecules, such as N_2 or NaCl, the outer or valence electrons must be assumed to describe much more complex orbits than plain ellipses, to account for the chemical bonds and observed spectra in terms of the quantum mechanics.

As a result of many years of painstaking research on the part of a number of prominent physicists, considerable information has been collected as to the probable number of electronic orbits and their characteristics, for practically all the known chemical elements. The principal source of such information is a study of spectral lines and groups of lines of such elements, interpreted in terms of the so-called quantum theory of elementary action.

The purpose of this article is to represent some of the results of these studies in the form of a graphical chart, and to describe an empirical rule of electronic orbits according to which elements seem to form periodic groups with recurrent

chemical properties. The data for the chart have been taken from various published tables (Ref. 1, 3, 6, 8, 10).

In the attached chart, the ninety two recognized chemical elements, from hydrogen to uranium, are marked as abscissas, under their respective atomic numbers. An atomic number indicates the number of extra-nuclear revolving electrons in an atom, and consequently also the net number of positive unit charges in its nucleus. While in general atomic numbers follow the same order as the atomic weights, the numerical relationship between the two is not quite simple. The recent tendency in physics is to recognize elements by their atomic numbers rather than weights.

Various kinds of orbits are indicated schematically by large circles, their meaning being stated along the axis of ordinates, in the manner customary in the quantum theory. Since it is not necessary to be familiar with this theory in order to understand the chart, the meaning of the notation may be briefly explained as follows:[1]

The symbols 1_1, 2_2, 3_3, etc., refer to *circular* electronic orbits of increasing diameters. The diameters do not increase as the numbers 1, 2, 3, but more nearly as their squares, because 1, 2, 3, refer to the numbers of units of so-called elementary action (energy times time, or moment of momentum times angle). For the purpose of this article it is sufficient to know that 1_1 is a circular orbit of an electron closest to the nucleus, and concentric with it; 2_2 is a concentric circular orbit of a much larger radius, etc.

The remaining symbols refer to *elliptic* orbits, the center of the nucleus always being one of the foci of the ellipse. The more nearly the subscript equals the principal number, the smaller the eccentricity of the ellipse. For example, the orbits, 4_3, 4_2, and 4_1 all have the major axes of the same length, equal to the diameter of the circle 4_4, but the minor axis of the orbit 4_2 is smaller than that of 4_3, etc. If an electron could simply oscillate about the nucleus, passing

[1] The quantum theory and its relation to spectroscopy has been the subject of so many books and articles, from very elementary to most advanced, that it is not practicable to give here even a brief bibliography of the subject. The reader will have to choose his own reading depending upon his aims and preparation.

through it, its orbit would be denoted by 4_0. The total "action" of an electron in a given elliptic orbit is considered to consist of two parts, that of rotation and that of changes in the length of the radius vector from the center of rotation (focus) to the electron. The number k which characterizes the first part of the action is known as the azimuthal number, the second as the radial quantum number. Thus, for example, for the elliptical orbit 4_3, three units of action correspond to rotation and one unit to changes in the length of the radius vector. The sum of the two, equal to 4, is known as the total quantum number, n.

The foregoing notation, of the form n_k, gives the total and the azimuthal quantum numbers. The radial quantum number can be found by subtraction. The orbits corresponding to the same total quantum number n are shown along the axis of ordinates combined into groups denoted by the letters K, L, M, etc. The historical reasons for this notation, in connection with X-ray spectra, need not be discussed here.

The chart is cumulative, that is, each chemical element possesses all the orbits of the preceding (lighter) elements and an added electron distinguishing it from its neighbor to the left. For example, an atom of iron (atomic number 26) possesses 26 revolving electrons in the following orbits:

Kind of orbit	1_1	2_1	2_2	3_1	3_2	3_3	4_1
Number of Electrons	2	2	6	2	6	6	2

In lieu of the subscripts 1, 2, 3, etc., to designate the azimuthal quantum number, letters s, p, d, etc., are often used, as indicated along the axis of ordinates. It will be seen that each letter always corresponds to the same subscript, as shown in the following table:

s	p	d	f	g	h	i
1	2	3	4	5	6	7

This, of course, does *not* mean that $s = 1$, $p = 2$, etc. As to historical reasons for the use of these particular letters, see any recent book on theoretical spectroscopy.

A classification of the atomic orbits in accordance with the values of n and k only is far from being complete. In accordance with the well established "exclusion principle,"

A CH

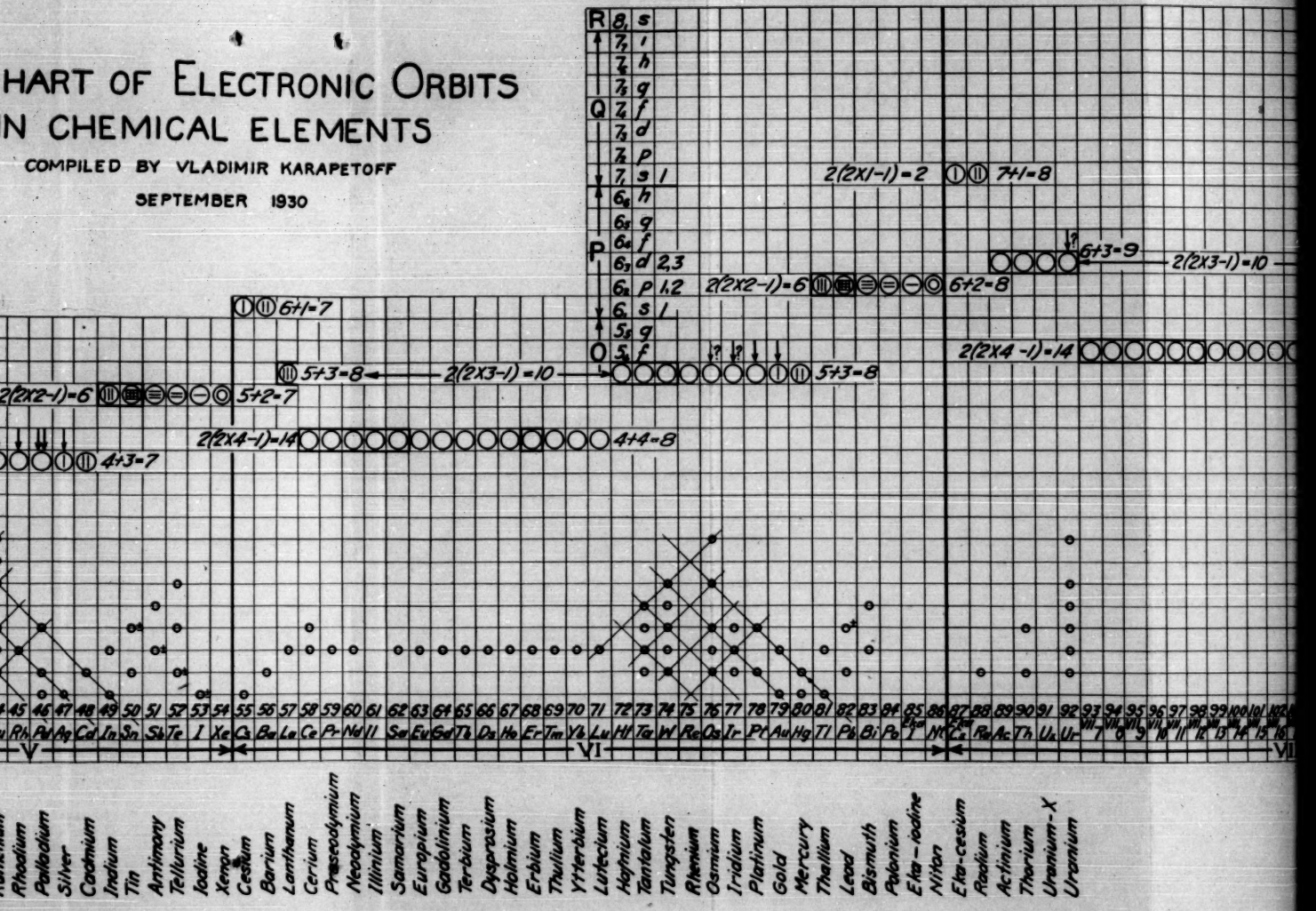
IN

Shell notation
Spectroscopic notation for k.
Third quantum number
Valence.
O
N
M
L
K
Valence
Atomic numbers
Symbols
Groups
2(2X1-1)=2
1+1=2
2(2X2-1)=6
2+2=4
2(2X1-1)=2
3+1=4
2(2X2-1)=6
3+2=5
2(2X3-1)=10
2(2X1-1)=2
4+1=5
2(2X3-1)=10
3+3=6
2(2X2-1)=6
4+2=6
2(2X1-1)=2
5+1=6
Hydrogen
Helium
Lithium
Beryllium
Boron
Carbon
Nitrogen
Oxygen
Fluorine
Neon
Sodium
Magnesium
Aluminium
Silicon
Phosphorus
Sulphur
Chlorine
Argon
Potassium
Calcium
Scandium
Titanium
Vanadium
Chromium
Manganese
Iron
Cobalt
Nickel
Copper
Zinc
Gallium
Germanium
Arsenic
Selenium
Bromine
Krypton
Rubidium
Strontium
Yttrium
Zirconium
Niobium
Molybdenum
Masurium
Ruthenium
Rhodium

CHART OF ELECTRONIC ORBITS IN CHEMICAL ELEMENTS

COMPILED BY VLADIMIR KARAPETOFF

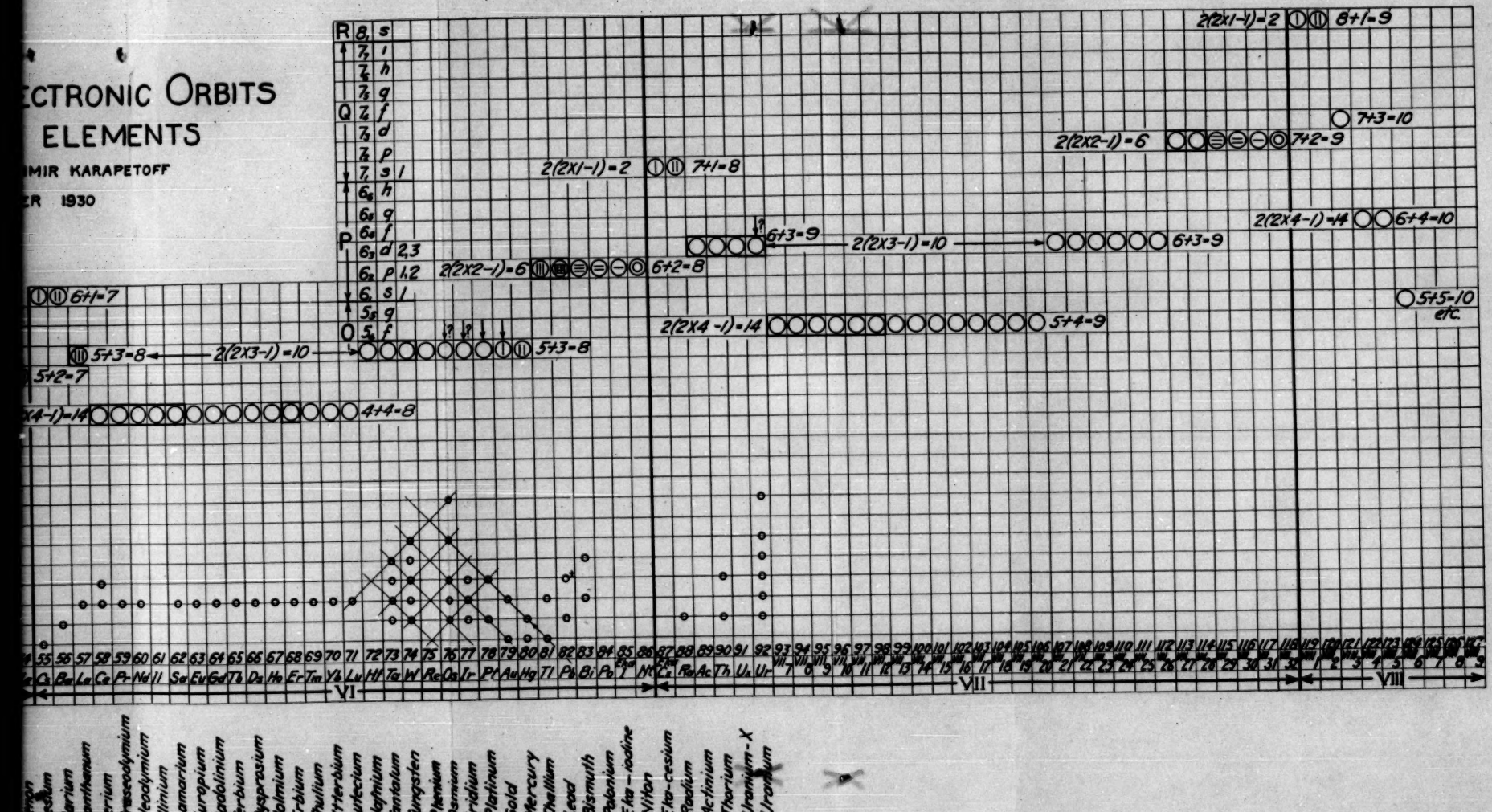
SEPTEMBER 1930



ECTRONIC ORBITS
ELEMENTS

MIR KARAPETOFF

ER 1930



Xenon
Cesium
Barium
Lanthanum
Cerium
Praseodymium
Neodymium
Illinium
Samarium
Europium
Gadolinium
Terbium
Dysprosium
Holmium
Erbium
Thulium
Ytterbium
Lutecium
Hafnium
Tantalum
Tungsten
Rhenium
Osmium
Iridium
Platinum
Gold
Mercury
Thallium
Lead
Bismuth
Polonium
Eka-iodine
Niton
Eka-cesium
Radium
Actinium
Thorium
Uranium-X
Uranium

no two electrons within an atom can have all their quantum numbers identical. For this reason, further quantum numbers have been introduced so that two or more electrons with the same values of n and k may still be discriminated. In the older geometric interpretation of the structure of the atom, with electrons thought of as small material corpuscles describing definite orbits, two or more identical orbits were imagined to lie in different planes passing through the center of the nucleus. This gave a basis for a further quantization of geometrically identical orbits by means of the vectors of their momenta.

In the light of the modern ideas of wave mechanics and electron spin, a somewhat different basis of quantization has been introduced. Nevertheless, for spectroscopic purposes, one may still speak of orbits such as 5_{32} and 5_{33} (five sub three-three) for both of which $k = 3$, but the third quantum number is 2 in one case and 3 in the other.

These second subscripts are indicated in the chart along the axis of ordinates next to the column with the letters s, p, d,. . . . An orbit such as 5_1 becomes 5_{11}, whereas the group of orbits 5_3 is split into 5_{32} and 5_{33}. This division is indicated in the chart by placing the orbits with the lower second subscript within a box. For example, there are two orbits 2_{21} and four orbits 2_{22}; the former are shown in a box.

The maximum number of electrons in a sub-group obeys the so-called Stoner rule (Ref. 2) which is as follows:

Quantum subscript	11	21	22	32	33	43	44
Max. no. of electrons	2	2	4	4	6	6	8

A further sub-division of the orbits is provided by the so-called "electron spin," a term which has become current, whether or not it adequately describes the physical phenomenon itself. An electron is imagined to rotate not only about the nucleus, but to spin about its own axis as well, somewhat as planets do. Thus, two electrons describing identical orbits can still be distinguished by the direction of their spin, the corresponding vectorial quantum numbers being $+ 0.5$ and $- 0.5$. This distinction has proved to be very useful, and in the chart the consecutive orbits in any sub-group may be thought of as provided with arrowheads

indicating rotation in the opposite sense. A further discussion of this topic is outside the purpose of this article and the reader is referred to theoretical treatises on quantum mechanics and spectroscopy. It has been merely desired to indicate a subdivision of the electrons in the chart into as many separate groups as possible.

As the number of electrons in an atom increases, the structure, from the dynamical point of view, rapidly becomes more complex, because of mutual repulsions among the revolving electrons; this is known in astronomy as perturbations. Thus, in reality the planets do not describe true ellipses about the sun but much more complex curves, due to varying attractive forces among the planets themselves. Similarly within the atom, as the number of orbital electrons increases, new combinations of stable equilibrium arise which under certain conditions lead to changes in some of the previous orbits.

Consider, for example, vanadium (23). It has two 4_1 orbits and three 3_3 orbits. The next element, chromium, should be expected to have two 4_1 orbits and four 3_3 orbits. In reality, spectroscopic evidence points to one 4_1 orbit and five 3_3 orbits. This means that the addition of a 3_3 electron causes one of the previous electrons in the "loose" or "penetrating" 4_1 orbit to change to a more closely bound 3_3 orbit. This fact is indicated in the chart by a vertical arrowhead pointing down, over the last added orbit. Similar changes or "irregularities" occur in a few other elements. In the case of palladium (46), both orbits 5_1 seem to change to 4_3, as indicated by two arrowheads. An arrowhead over a circle always occurs on a *d*-orbit and means that one of the preceding two *s*-orbits of the higher total quantum number n must be changed to an identical *d*-orbit. For example, Ux (91) has two 7_1 electrons and three 6_3 electrons. The next element, Ur (92), is marked with an arrowhead. This means that instead of two 7_1 electrons and four 6_3 electrons, its spectrum corresponds to one 7_1 electron and five 6_3 electrons. A question mark indicates insufficient or contradictory evidence.

In the elements 25 to 28 arrowheads pointing up are shown within the circles. These indicate that one of the orbits in the preceding boxed sub-set must be raised to the

next sub-level. More specifically, the elements 25 to 28 possess only three 3_{32} orbits instead of four shown in the box, the fourth one being changed to 3_{33}. Thus, cobalt (27) has three 3_{32} orbits and four 3_{33} orbits, whereas without the arrowhead the chart would indicate four 3_{32} orbits and three 3_{33} orbits.

The total number, N, of orbits in a given horizontal set of circles depends only on the azimuthal quantum number, k; the relationship between the two being [2]

$$N = 2(2k - 1). \qquad (1)$$

This equation is written in the chart to the left of each horizontal set of orbits. The actual numbers are shown in the following table:

Kind of Orbit.	Maximum Number of Electrons.
s	2
p	6
d	10
f	14

There is a certain periodicity in the arrangement and number of orbits, pointed out by Bohr. For elements with low atomic numbers, the periodicity is the same as in the horizontal rows of the familiar Mendelejeff table; for heavier elements the new periodicity is more natural and logical. Each "row" of elements ends with an inert gas, such as helium (2), neon (10), argon (18), etc. These gases are marked with double circles, to bring out the periodicity more clearly. Each row begins with an alkali metal, such as lithium (3), sodium (11), potassium (19), etc. The rows are denoted by Roman numbers below the symbols for the elements.

The elements just preceding the inert gases belong to the halogen family: fluorine (9), chlorine (17), etc. Thus, a halogen, an inert gas, and an alkali metal, occupy three adjacent places at the transition points between the consecutive rows of elements. The last orbits of the halogens and inert gases always belong to a group of six characterized by the azimuthal quantum number 2 (p-orbits). The next

[2] This relationship is explained below; see also McLennan, Ref. 7.

alkali metal is distinguished by a single orbit of azimuthal number 1 (an *s*-orbit, a very elongated ellipse in heavier elements) and of total quantum number exceeding that of the preceding electrons by 1. Six orbits of azimuthal number 2 seem to form a limit of stability, resulting in an inert gas. Because a halogen lacks one of such electrons and an alkali metal has a loosely coupled or penetrating electron, may be the reason for a strong chemical affinity between these two kinds of elements.

THE n-PLUS-k RULE.

Following the chart, from elements of low atomic numbers to those of high atomic numbers, new groups of electronic orbits do not appear in the same order in which they are marked on the axis of ordinates. For example, the last orbit 4_3 occurs in the element 48, and the first orbit 4_4 does not appear until the element 58, after some orbits with the total quantum numbers 5 and 6 have made their appearance.

An empirical rule governing the order in which different kinds of orbits appear, is shown in the appended Table A. In

TABLE A.

Sequence of Orbits.

	Total quantum number n.	Azimuthal quantum number k.	$n + k$.	$n - k$.
$(m-1)$st row of elements	↑ etc. $m-1$	↑ 2	↑ $m+1$	↑ $m-3$
mth row of elements	m	1	$m+1$	$m-1$
	$m-1$	3	$m+2$	$m-4$
	$m-2$	4	$m+2$	$m-6$
	$m-3$	5	$m+2$	$m-8$
(m even)	$0.5m+1$	$0.5m+1$	$m+2$	0
(m odd)	$0.5m+1.5$	$0.5m+0.5$	$m+2$	1
	$m-3$	5	$m+2$	$m-8$
	$m-2$	4	$m+2$	$m-6$
	$m-1$	3	$m+2$	$m-4$
	m	2	$m+2$	$m-2$
$(m+1)$st row of elements	$m+1$ ↓ etc.	1 ↓	$m+2$ ↓	m ↓

this table, m denotes the serial number of a horizontal row of elements (not a Mendelejeff vertical group), such as are marked by Roman numerals along the axis of abscissas in the chart; n is the total quantum number of an orbit, and k is its azimuthal number. The table indicates the order in which the groups of orbits appear in the elements of the mth row, together with the last orbits of the preceding row, $m - 1$, and the first orbits of the following row, $m + 1$. The next to the last column in the table gives the values of $n + k$ which remain constant for a row, except for the orbits with the azimuthal number 1. The values of $n + k$ are also written to the right of each set of orbits in the chart, partly to indicate that this sum remains constant, and partly to make it easier to read the quantum numbers of the set. Thus, if the formula reads $4 + 3 = 7$, it means that the set corresponds to the orbits 4_3, and it is not necessary to refer to the axis of ordinates in order to find the values of n and k for that set of orbits.

The n column in the table is formed by beginning with $n = m$, where m is the row number. Integers are then written down, each smaller than the preceding one by 1, until the value $0.5m + 1$ is reached for m even, or $0.5m + 1.5$ for m odd. Then the numbers are continued in an increasing order until the value m is reached again. This gives all the possible values for n in that particular row.

The column for $n + k$ can be filled out next, by writing $m + 1$ in the first line and $m + 2$ for the remainder. The k column can then be filled out by subtraction. It will be seen that the values of k begin with 1; the azimuthal number 2 is omitted. The numbers then increase by 1 up to $0.5m + 1$ for m even and to $0.5m + 0.5$ for m odd, and then decrease again to 2.

All the known elements seem to obey this rule; for this reason hypothetical unknown elements or mono-nuclear combinations, up to the atomic number 124, are also shown in the chart at the extreme right, in accordance with this rule. Uranium (92) being the heaviest known element, those beyond it are denoted by the row number and by the serial number in the row, for example VII7, VII8, etc. This extension of the chart is not necessarily a prophecy, since

even the heaviest known elements are quite unstable (radioactive). The chart has been extended partly to illustrate the rule given in the table, partly to test that the rule does not give any mathematically absurd results, and partly to indicate that some kinds of orbits, such as 5_4 and 6_3, which do not occur in any of the known elements, might be realisable with heavier nuclei. In fact, all the possible kinds of orbits, indicated along the axis of ordinates, are covered by the proposed rule. Perhaps knowing which orbits to expect, may help in a search for new mono-nuclear systems beyond uranium.

Since n is the total quantum number and k is its azimuthal part, k cannot be greater than n. This explains the lowest figures, $0.5m + 1$, etc., in the table. For m even, the two are equal, giving a circular orbit; for m odd, n is greater than k by 1.

In application to the first row, consisting of hydrogen and helium, the whole Table A reduces to $n = 1$ and $k = 1$. No other values are possible. For the second row, the table becomes

n	k
2	1
2	2

these being the first and the last horizontal lines of the general scheme. No intermediate lines are possible because k would be greater than n. This shows that in the second row of elements, from lithium to neon, the elliptical orbits 2_1 appear first, followed by the circular orbits 2_2.

For the third row the table gives

n	k
3	1
3	2

being again the first and the last lines of the general table. This is of interest showing that circular orbits 3_3 are excluded from this row. It is left for the reader to apply the table to the remaining rows and to show that the results check with the order of the sets of orbits indicated in the chart. We show below the application of the table to the seventh row:

8	k
7	1
6	3
5	4
6	3
7	2

An interesting feature of this case is that the orbits 6_3 occur twice, before and after the orbits 5_4. The total number of 6_3 electrons being $2(2 \times 3 - 1) = 10$, four orbits are shown in the chart preceding the orbits 5_4 and six following them. This division is somewhat arbitrary in that it refers to unknown elements. One reason for this division is that uranium being the last known element some change in the constitution of the atoms following it might be expected. The other reason is that the maximum number of 6_{32} electrons, according to the above mentioned Stoner rule, is four, so that a new sub-group or group must begin immediately after uranium.

A similar split set of orbits occurs in row VI, one 5_3 orbit preceding the 4_4 orbits and nine following it. This existence of a split set in a known row of elements gives more validity to postulating a similar split set in the next row, in the region of unknown elements.

The rows VI and VII have four different kinds of external orbits, and being the most developed or complete rows, a nomenclature of orbits suitable for these rows will be also suitable for the rows comprising elements of lower atomic numbers. The following notation is proposed:

Name.	Quantum Symbol.	Type of Orbit.
The first pair	m_1	s
The split ten—left	$(m-1)_3$	d
The bottom fourteen	$(m-2)_4$	f
The split ten—right	$(m-1)_3$	d
The final six	m_2	p

In rows V and IV the "bottom-fourteen" orbits are missing and consequently the ten are not split. In rows III and II the only orbits present are the first pair and the final six. The first row has the first pair only. Thus, the notation proposed for the most complex row is adequate for the simpler rows.

THE $2(2k-1)$ RULE.

The rule described in the foregoing section gives the consecutive order in which various sets of orbits appear as the atomic numbers increase, but it gives no information as to the limiting number of orbits of a given kind, that is, the number of electrons characterized by the same values of n and k. The latter problem has been solved on the basis of the following two assumptions:

(a) An orbital electron within an atom is completely determined by four separate quantum numbers. The third quantum number, say q, determines a discrete position of an orbit in space, that is, the vectorial position of the normal to the plane of the orbit. We shall assume that q may have any integral value, positive or negative, not greater than $k-1$, or be equal to zero. For example, if $k=2$, q may be equal to $+1$, 0, -1. Generally speaking, then, the total number of different values of 2 corresponding to a given k is

$$2(k-1)+1=2k-1. \qquad (2)$$

The fourth quantum number refers to the electron spin. Namely, each orbital electron is assumed to be spinning either clockwise or counter-clockwise about a certain axis passing through the center of its charge. To designate this distinction, the quantum number $+1/2$ or $-1/2$ is assigned to each.

(b) In an atom, no two orbital electrons may have the same values of all the four quantum numbers (Pauli's exclusion principle).

Combining the statements (a) and (b), it will be seen at once that the total number of circles in a horizontal row in the chart is $2(2k-1)$. Pauli's exclusion principle forbids more than 2 orbits having the same values of n, k, q; one of these orbits has an electron spinning say clockwise $(+1/2)$ and the other counter-clockwise $(-1/2)$. The total number of possible values of q (with a given k) is $2k-1$, and this number is multiplied by two on account of the electron spin.

THE NUMBER OF ELEMENTS IN A HORIZONTAL ROW OF PERIODIC SYSTEM.

We shall make the assumption that an inert gas is reached (and consequently a new row begins) when a p-set of electrons

has been completed. This assumption, together with the $(n + k)$ Rule and the $2(2k - 1)$ Rule explained above, permit to construct the whole chart of elements theoretically. The $(n + k)$ Rule determines the order in which various sets of orbits appear, and the $2(2k - 1)$ Rule tells the number of electrons per set. Having thus constructed the chart, it remains to locate the ends of the p-sets of orbits, and these will determine the points of division of the elements into horizontal rows. These points will also determine the vertical groups in the usual form of Mendelejeff's periodic table. The Stoner rule enables us to divide each set of orbits into two sub-sets.

CHEMICAL VALENCE.

It is supposed that the electrons in the outermost orbits account for the chemical valence of an element. Thus, alkali metals (3, 11, 19, etc.) have but one electron in the outer 'shell.' Moreover, this electron revolves in an s-orbit which, by assumption, has the greatest possible eccentricity. With the nucleus as one of the foci, the electron is therefore at a considerable distance from the nucleus during part of each revolution and may be said to be loosely bound. In other words, under proper conditions such an atom readily gives up this electron and becomes positively charged. In chemical reactions alkali metals act as univalent elements, the loose electron acting as a single bond. A single valence of this kind is denoted in the chart by a single vertical line inside the corresponding circle.

The halogens on the other hand (9, 17, 35, etc.) have five electrons in the external p-shell. Six orbits being the stable or neutral limit, such elements readily annex an extra electron under proper conditions. Thus, halogens are also univalent, but electronegative, and this may explain their strong combining power with the alkali metals. In the chart, the halogens are indicated by a single horizontal line within the circle.

The expressions "to annex an electron" or "give up an electron" should not be understood quite literally in the formation of molecules. The present view seems to be that compounds and molecules of polyatomic elements are essentially formed by sharing electrons. For example, in the hydrogen molecule, H_2, we are to think of the two atomic

nuclei as placed a certain distance apart and the two electrons describing symmetrical orbits about them in such a manner as to surround both nuclei. Such electrons are called shared electrons.

Similarly, in a molecule of sodium chloride, NaCl, we need not think of the atom of sodium as actually having given up its valence electron altogether to complete the outer shell of the chlorine atom. The valence electron may be imagined as being shared by the two atoms, simultaneously giving a semblance of the stable neon structure to the atom of sodium and of the stable argon atom to the chlorine atom. Of course, when NaCl is dissolved in water, chlorine atoms are supposed to possess an extra electron each and act as free negative ions, whereas positive sodium ions are sodium atoms minus their valence electrons.

Thus, between a symmetrical sharing of electrons in a diatomic molecule of an element and a complete separation in an electrolyte, there seem to exist all gradations of sharing. One may think of two or more persons owning some property in common, or at least using it in common, and the various degrees of possession (in kind or in relative duration of time), with equal use by all as one limiting case and one of owners completely ceding all his rights as the other limiting case.

Bivalent and trivalent elements of the second and the third vertical group (in the usual Periodic Table) are marked by two and three vertical lines respectively, whereas the electronegative elements of the fifth and sixth groups are distinguished by three and two horizontal lines. These lines are to be interpreted as the numbers of electrons which these elements are likely to give up or to acquire respectively.

The intermediate elements, such as carbon (6), silicon (14), etc., which form the fourth group, are quadrivalent, and are marked by four vertical lines and four horizontal lines within each circle. This is done to indicate that these elements, in forming compounds, may either acquire or lose four outer electrons. Until the nature of the chemical bonds in these elements is better understood, it is preferred to leave both the vertical and the horizontal lines in the valence symbol of such elements. In assuming these elements to be quadrivalent and electropositive, we postulate of course

that not only the two electrons in the outermost shell may be shared, but the two immediately preceding ones as well.

Quite a number of elements manifest different valences in different compounds. For example, nitrogen is trivalent in compounds with hydrogen and metals, whereas it is frequently quinquivalent in compounds containing oxygen and other negative elements. Sulphur, copper, chromium, iron, etc., also show alternative valences. To explain these, it may be assumed that these elements are capable of giving up not only their outermost electrons, but some of those in the next shell as well. Again, an incomplete shell of electrons may be either completely given up or else the atom may acquire a sufficient number of electrons to complete the shell. In the first case it acts as an electropositive element and in the second case as electronegative. Here again, neither the surrender nor the acquisition need be complete. It is sufficient to think of the electrons in question as coming more within the range of the influence of a second atom, or on the contrary some electrons from the second atom coming within the range of the atom under consideration. For details see Refs. 5, 8, 9.

For many elements, the valence shown within the circles is only one of the possible ones with the assumed structure of the atom. For heavier elements, remote from one of the inert gases marked with the double circle, the valences do not seem to depend in an evident manner upon the position of the element. It is possible to picture this fact by assuming that with ten or fourteen almost identical orbits, two or more slightly different stable configurations may co-exist. For such elements the rest of the structure of the whole atom probably has to be considered, and not only the two outer shells, in order to connect the valences with the position of the element in the chart.

The valences are also shown above the axis of abscissas by small circles. It will be seen that quite a number of elements possess multiple valences. Unless otherwise indicated, the valences are positive, that is, they represent a tendency on the part of the atom to give up some of its electrons, becoming a positive ion. Negative valences are indicated by the minus sign, and where an element happens

to possess equal positive and negative valences the symbol ± is used.

Some diagonal lines are shown over the valence circles between the elements 37 and 47. The purpose of these lines is to indicate by their intersections the possible unknown valences of the element 43. A similar construction is shown for the element 75. The values of the valences indicated in the chart are those given by Segerblom (Ref. 11) and may differ in some particulars from those of other authors.

Isotopes, or modifications of the same element, having the same atomic number but different atomic weights, are not indicated in the chart because the relationship between their presence and the position of the element in the chart is not yet clear. Of the rare gases, neon (10) and argon (18) have two isotopes each, krypton (36) has six, and xenon (54) has at least seven (Ref. 4).

There is a decided tendency towards formation of isotopes in the elements just preceding the final six of each row. Mercury (80) has eight isotopes, cadmium (48) has six, zinc (30) has four, and magnesium (12) has three. Accordingly, beryllium (4) might be expected to have two isotopes, but in reality it is the next element, boron, that has two.

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ITHACA, NEW YORK.
September, 1930.

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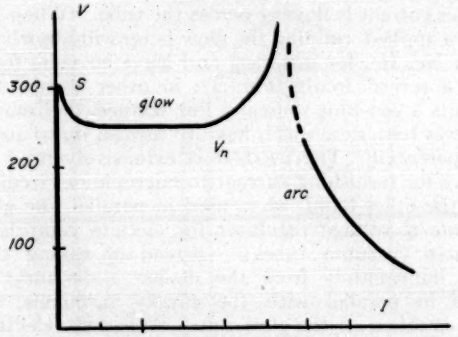
THE USE OF DISCHARGE TUBES IN ELECTRIC CIRCUITS.

BY

RICHARD RUEDY, D.Sc.

The factors which give valuable properties to an electric discharge through a gas at low pressure are (*a*) the rapid increase in current inside a narrow range of voltages, in the neighborhood of the sparking or starting potential "*S*" (from the Townsend current to the glow discharge and to the arc) (Fig. 1), (*b*) the development, despite varying current

FIG. 1.



The different stages of a discharge through a gas at reduced pressure (ionization current up to *S*, then self-sustaining discharge).

strength, of a constant potential drop between two electrodes, placed close together, and of sufficient size, (*c*) the rapid increase of the potential drop with increasing current when small electrodes are used (abnormal cathode fall), (*d*) the violent agitation of the negative charges corresponding to a value far above that of the thermal energy of the gas molecules in the tube, so that the electrons are able to reach, and charge, a small insulated metal body to a potential eventually equal to that through which they have fallen; when this point is

reached, further influx of electrons is stopped in virtue of the charge acquired by the small body. These are all in the first line self-adjusting properties, changing between fairly sharp limits, so that the discharge tube can become of great help in problems of automatic controlling circuits. The drawbacks of the discharge tube are the fairly high potentials which are necessary for its functioning, and the series resistance which must always be included when running a discharge tube, in order to prevent the current from running away to high values (arc).

VOLTAGE REDUCERS, REGULATORS AND RECTIFIERS.

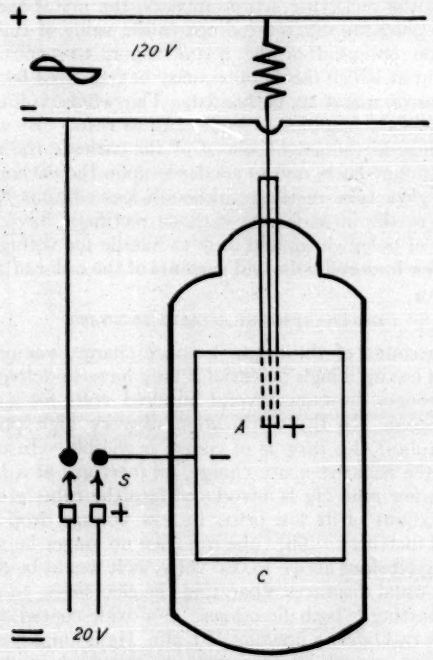
So long as the electrodes are not completely covered with the discharge, and are not too far apart, a glow tube works at a constant potential equal to the normal cathode fall V_n whatever current is flowing across the tube. When 100 volts D.C. are applied, running the glow lamp will absorb $V_n = 80$ volts in Ne, He, for instance, and leave 20 volts for further use in a series circuit (Fig. 2); in other words, the tube represents a constant source of low voltage, and may replace dry or wet batteries, which have to furnish up to 400 mA. at a low potential. The method is extensively used in some countries for furnishing current to microphone circuits.

On the other hand, when used in parallel, the glow tube represents a voltage regulator for circuits comprising high resistances (vacuum tubes). Instead of taking the D.C. tension immediately from the mains, a discharge tube is inserted in parallel with the supply terminals, and the voltage is taken off the glow tube. When the electrodes are sufficiently large, the discharge potential remains constantly at V_n, and represents a more invariable source than the mains.

If A.C. only is available, it can be rectified in the tube by making use of the properties of the abnormal cathode fall. A glow tube having 2 dissimilar electrodes, in the first place of different size, can be used, one electrode being, a large disk, or cylinder C, for instance, the other a small bead A. The disk will carry a current of many mA. at the normal voltage V_n; but when the bead has to serve as a cathode, it is able to carry but a small current before the potential rises rapidly

above the normal cathode fall. On connecting the tube to a source of alternating voltage, in series with a resistance, the current may have a value of 50 or 100 mA. in one direction depending on the size of the large electrode, and be negligible in the second half of the cycle, when the bead becomes cathode,

FIG. 2.



The discharge tube as a voltage reducer.

at least if the applied potential does not exceed a few hundred volts. Such tubes are widely used in batteryless radio receivers for rectifying the A.C. supply (Raytheon tubes), and are also on the market for transmitter tubes in sizes giving up to 300 mA. at 2000 to 3000 volts.

The efficiency of the rectifier can be improved by using for the large electrode a metal with a low cathode fall, K for instance. The tube becomes then still more pervious in the direction for which K is cathode, due to the lowering in the values of both S (starting potential) and O (operating potential). With increased current density, a third factor favoring the rectifying action appears, the metal evaporates and the potential difference drops to the value of that of the ionization potential of the metal vapor, to 3 or 4 volts. The point at which this happens may be regulated by altering the mass of metal to be heated. The whole voltage drop across the tube amounts to from 12 to 18 volts. By applying guard rings in the neighborhood of the cathode the metal is caused to flow back, and to condense upon the cathode.

The glow tube rectifiers, although less efficient than the Hg arc, or the incandescent cathode rectifiers, have the advantage of being cheap and easy to handle for voltages from 10 to a few hundred volts and currents of the order of 200 mA. and below.

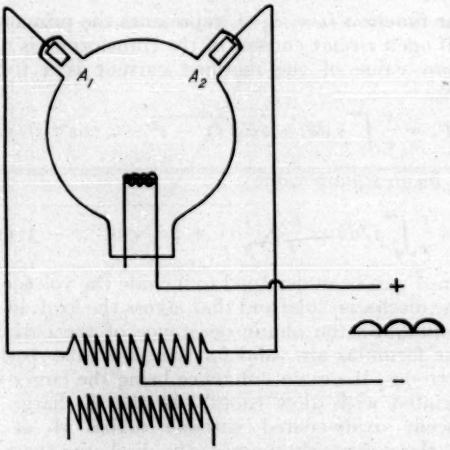
GAS-INCANDESCENT CATHODE RECTIFIERS.[1]

On account of the negative space charge, vacuum tube rectifiers use up a high potential if they have to deliver somewhat stronger currents, several hundred volts for a current of 1 ampere. In the rectification of very high potentials (X-ray tubes) this drop is of course negligible.—In order to destroy the negative space charge, an inert gas at a pressure of 1 or a few mm. Hg is introduced into the tube, preferably A, on account of its low price, its low voltage drop and its chemical inertness. The tube can then no longer be used for rectifying tensions above 10,000 volts, as it would be difficult, with the usual distances separating the electrodes, to prevent it from starting in both directions. For oxide-coated cathodes (Wehnelt cathodes) a pressure of 1 mm. Hg is employed, for a W filament several cm. pressure are required in order to slow down the destruction of the filament caused by evaporation, and the gas has to be free from chemical impurities such as O_2 (Tungar rectifiers). With A the drop at the cathode amounts to but a few volts and the space charge is small even

[1] Guenther Schulze, Elektrische Gleichrichter, 2d ed., 1929.

for currents of over 20 A. The main loss occurs at the anode where drops, equal to the ionization potential of the gas, or 16 volts, are found. The total voltage used up in the bulb may be from 16 to 18 volts, and it remains constant over the range of a fraction of 1 A. to 30 and more A. The starting voltage in the direction anode to filament can be calculated from Townsend's equation, and depends, of course, on the pressure of the gas, the shape and the distance of the electrodes. The rectifiers with W filaments are usually built with one anode only, the tubes working with Wehnelt cathodes are manufactured with 2 anodes so that both half-waves are rectified. To this end the secondary of the transformer is divided into 2 halves (or, more sections in the case of polyphase currents), the middle point of the windings functioning as the negative terminal of the rectified current (Fig. 3).

FIG. 3.



Gas-incandescent cathode rectifier.

The positive potential is picked up from some point of the filament circuit, preferably from the center tap of a resistor shunting the filament. The primary voltage being

$$e = E \sin pt = E \sin \theta,$$

the voltage induced in the secondary will be

$$e_{s1} = E_s \sin \theta,$$
$$e_{s2} = -E_s \sin \theta,$$

where $E_s = zE$.

In the first half of the cycle, A_1 will therefore serve as anode, in the second half A_2. There will be gaps in the rectified current, because no discharge is possible so long as e_s is smaller than the necessary voltage drop A, and the current is determined by

$$i_r = E_s \sin \theta/r - A/r = E_s(\sin \theta - s)/r$$

in the second half by

$$i_r = -E_s(\sin \theta + s)/r.$$

A similar function $i_n = \pm zi_r$ represents the primary current drawn, if open circuit current of the transformer is neglected. The mean value of the rectified current is a function of $s = A/E_s$,

$$I_m = \frac{1}{\pi}\int_0^\pi i_r d\theta = 2E_s(\sqrt{1 - s^2} - s \cos^{-1} s)/\pi r,$$

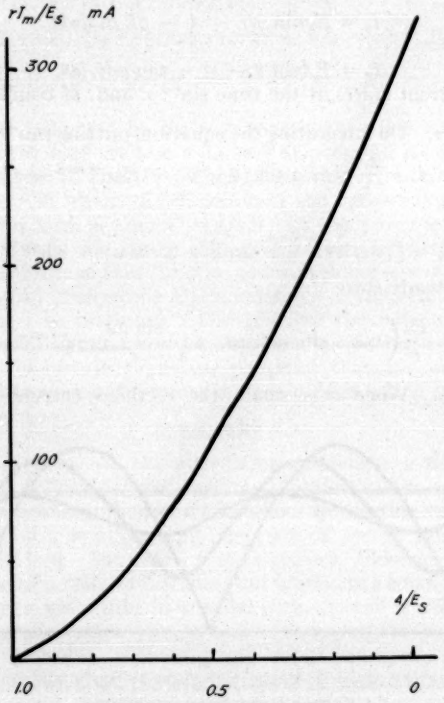
the root mean square value

$$I_{rms} = \frac{1}{\pi}\int_0^\pi i_r^2 d\theta = \frac{E_s}{r}\sqrt{\frac{1}{\pi}(1 + 2s^2) \cos^{-1} s - 3s\sqrt{1 - s^2}}.$$

The term A can be understood to include the voltage drop V_n across the discharge tube and that across the load; in a similar way r represents the ohmic resistance of the entire circuit. The same formulas are valid for the glow tube rectifier with cold electrodes, the main difference being the larger values of V_n associated with glow tubes. In the discharge from an incandescent oxide-coated cathode across A at reduced pressure, the voltage drop across the discharge space remains at an almost constant value of 18 to 16 volts, when the current is made to increase from a fraction of an A. to 50 and more A. In the case of $E_s = 160$ volts (peak value), $r = 100$ ohms, $A = 16$ volts, therefore $s = 0.1$ and $\cos^{-1} s = 1.47$, the mean

value of the pulsating rectified current becomes $I_m = 0.85$ A. (Table I and Fig. 4).

FIG. 4.



Mean value per anode of rectified current as a function of A/E_s.

TABLE I.

Rectified Current I_m in mil A.

A/E_s =	0.6	0.5	0.4	0.3	0.2	0.1	0
rI_m/E_s =	77	109	144	185	224	269	319

The pulsating rectified current can be smoothed out by

adding inductance in the D.C. circuit: when the cathode is connected with L in series with r, then from the theory of the Hg arc rectifier [2, 3]

$$i_r = E_s \sin \theta/r - A - pL_c di_r/rd\theta$$

or

$$i_r = E_s(\sin \theta - s) - \tan a di_r/d\theta.$$

The current starts at the time $\sin^{-1} s$, and, of course, $\sin^{-1} s < \theta \leqq \pi$. On integrating the equation, putting $\tan a = pL_c/r$

$$i_r = \frac{E_s}{r}[\cos a \sin (\theta - a) + H_0 \epsilon^{-\theta \cot a} - s]$$

with

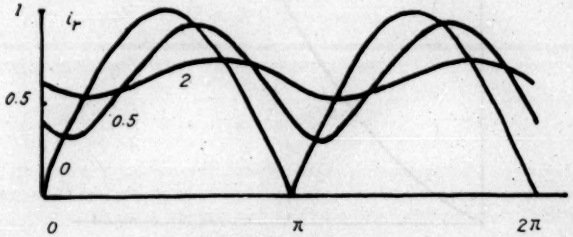
$$H_0 = [(\cos a \sqrt{1 - s^2} \sin a - s \cos a) + s]\epsilon^{\sin^{-1} s \cot a}$$

in the steady state

$$i_r = \frac{E_s}{r}\left(\cos a \sin (\theta - a) + \frac{\sin 2a}{1 - \epsilon^{-\pi \cot a}} \epsilon^{-\theta \cot a} - s\right)$$

(Fig. 5). When s is small, the rectified current becomes

FIG. 5.



Wave form of rectified current for different values of $\tan a = pL/r$; with increasing "a" the peaks are smoothed out and displaced.[2]

entirely constant for $L = \infty$, and

$$i_r = E_s(2/\pi - s)/r = I_{m\infty}.$$

Whenever the cathodic induction coil is fairly large, the

[2] L. B. W. Jolley, Alternating Current Rectification, 3d ed., 1928.
[3] K. E. Mueller, Quecksilbergleichrichter, 1925.

rectified current can be represented by

$$i_r = I_m - c \sin (2\theta + (\pi/2 - a)),$$

which developed into a Fourier series gives

$$i_r = (2/\pi - s)E_s/r - 0.22 \sin (2\theta + \pi/2 - a) \times E_s/pL$$
$$= \frac{E_s}{pL}[(0.637 - s)pL/r - 0.22 \sin (2\theta + \pi/2 - a)].$$

The power delivered by the rectifying current depends upon the load; the load may be a battery of potential E_b which is to be charged, so that $P = E_b I_m$ (neglecting A), or an ohmic resistance r in which the direction of the current is not important; in such a case $P = rI_m^2$. In the power packs of radio receivers, the choke coils have values $L = 10$ henry, $r = 2000$ ohms, so that for $p = 300$ we get $\tan a = 1.5$. By using several choke coils and condensers a perfectly smooth current can be produced. The effect of the inductance on the cathode side can also be obtained by placing a coil in each of the anode branches, provided that the coils are coupled together and produce magnetic fields which have the same direction.

RELAYS.[4, 5]

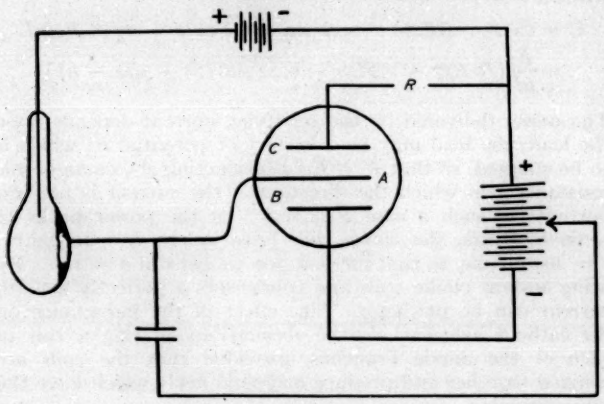
The property of the glow lamp to deliver a negligible current closely below the starting potential; and to multiply its intensity many thousand fold when the starting voltage is exceeded by a small amount, suggests its use for relays and safety devices. In case of overtensions endangering the insulation of a coil, for instance, but involving a small amount of energy, a glow tube in parallel with the coil is capable of absorbing the excess energy, and of lowering the tension to the normal value, without itself being destroyed. At the same time the glow reveals the existing high voltages. In this connection, a glow tube is very reliable as a protection in aërials: when shunting the earthing switch, it is capable of absorbing high tensions and of protecting therefore the receiving instruments.

In the case where only very little energy is available in a circuit AC, a third electrode B may be connected with one

[4] H. Richter and H. Geffcken, *Zeitschr. techn. Physik.*, 7, p. 601 (1926).
[5] R. T. Wilkins and F. B. Friend, *J. Optical Soc. Am.*, 16, p. 370 (1928).

of the main electrodes, A or C, over a battery sufficient to deliver 20 to 50 mA. (Fig. 6), the tension being kept slightly

FIG. 6.



The discharge tube as a relay.

below the starting potential, so that normally no current will flow in this circuit. When now a very small current starts to circulate in the main circuit AC, of the order of 10^{-10} A. or less, the high mobility of these charges at the low pressure existing in the glow tube allows a positive space charge to be built up between the electrodes AB of the auxiliary circuit. The starting potential in this circuit is lowered, on account of the distortion of the field produced by the positive ions, and a current of 10 to 50 mA. is delivered. This system of relay may be used for announcing very small changes in a photo-electric cell or in a radio receiver, or changes in the conductivity of gases, liquids, etc. It may be improved by adding C and L in order to make the circuit self-interrupting; the current will then only flow as long as the changed conditions last. The use of this relay in observatories, for the reception of time signals, has shown that its lag is smaller than 1/100 sec. The high sensitivity of the glow tube relay becomes apparent when the main circuit AC is opened, and

the positive pole of the battery earthed. The charge communicated to the electrode *A* by touching it is then sufficient to cause breakdown and flow of some 20 mA. Leakages through high resistances or across condensers, or induced by vibrations of vehicles and trains may conveniently be studied in this way.

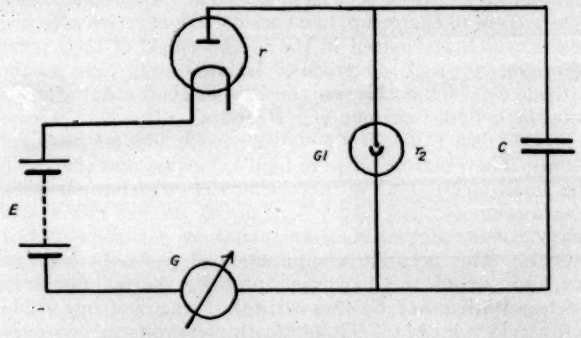
Somewhat stronger discharges can be used to start an arc discharge at reduced pressure. The *W* arc and point-lamp are started by exciting a small discharge between *B* and *A*, where *B* is a Mg electrode, for instance, which works at 110 to 220 volts. Once the gas is ionized, between *A* and *B* the arc begins to form between the electrodes *A* and *C*, as this latter circuit is constructed with lower resistance.

Another type of relay with 3 electrodes is based upon the high velocity of the electrons produced at the cathode, which allows them to charge up, to a considerable negative potential bodies which are placed in the neighborhood of their point of origin. A third electrode *B* is introduced, close to the cathode *C*, which in this case consists of a short rod; the anode being a cylinder surrounding *B* and *C*. The field is concentrated near *C*, and if it is strong enough, ions are produced there. The electrons acquire high velocities, and charge up the small electrode *B* to a considerable negative potential. The motion of the positive ions across the gas meets with heavy losses, so that they are unable to neutralize *B* fast enough. The negative charge upon *B* prevents the free development of the ionization process, and no discharge starts. With *B* and *C* connected together, a discharge would pass at 350 volts, but if *B* is left floating 900 volts are necessary to produce a measurable current. But when *B* is connected to a large capacity, or when its charge may leak off across a high resistance, such as an ionization chamber or a photoelectric cell, the obstacle to the discharge between *C* and *A* is removed, the discharge starts and indicates that current is flowing in the photoelectric cell or in the ionization chamber. —In the case of Hg and incandescent cathode rectifiers, in which the cathode comes close to the walls, the glass walls assume sometimes the rôle of the third electrode *B*.

GLOW TUBE BUZZERS.[6, 7, 8]

When a large condenser "C" is put in parallel with a glow tube or with its series resistance, the source of potential has to charge up the condenser (and the electrodes) across the resistances present in the circuit. Once the starting potential S is reached, the condenser discharges rapidly across the tube, the potential drops until the normal cathode fall O is established. The charging current must now be strong enough to maintain both the electrodes and the condenser at this potential; else the discharge current decreases to a low value, and eventually stops. When the starting potential is again reached, a new cycle begins. The charging current is evidently $(E - V)/r$ if V is the potential of the condenser, and (Fig. 7)

FIG. 7.



Glow buzzer circuit.

$$i = (E - V)/r = CdV/dt$$
$$dt = -CdV/(V - E).$$

Integrated between the limits S and O, the equation gives for the time t' necessary for charging

$$t' = Cr\ln\frac{E - O}{E - S}.$$

[6] W. Schallreuter, Schwingungserscheinungen in Entladungsroehren Samml. Vieweg 66.

[7] J. Taylor and W. Clarkson, *J. Scientific Instrum.*, I, p. 173 (1924).

[8] W. A. Leyshon, *Phil. Mag.*, 4, p. 305 (1927).

At the potential S the discharge starts, and the potential of the condenser drops during the interval t'' to the value O. During this time there are 2 circuits to be considered. If the average resistance of the discharge path is r_2, so that V/r_2 is the current corresponding to the potential V of the condenser, $i = (E - V)/r$ and $i_1 = CdV/dt$ (condenser current)

$$CdV/dt + V/r_2 = i = (E - V)/r,$$
$$Crr_2dV/dt + V(r + r_2) = Er.$$

On integrating

$$V = Er_2/(r + r_2) + k\epsilon^{-(r+r_2)t/Crr_2}$$

or

$$t = \frac{Crr_2}{r + r_2} lnE \cdot \frac{r_2}{r + r_2} - \frac{Crr_2}{r + r_2} lnV + \frac{Crr_2}{r + r_2} lnk.$$

Therefore the time in which the potential varies from S to O

$$t'' = \frac{Crr_2}{r + r_2} ln \frac{Er_2 - S(r + r_2)}{Er_2 - O(r + r_2)}.$$

When r is sufficiently large

$$t'' = Cr_2 lnS/O.$$

The value of r_2 representing the discharge path is small, a few thousand ohms during the passage of the discharge, and therefore

$$t'' \ll t'.$$

For a given value of C and r, t' will be small for very large E, as is to be expected; it will become very large, on the other hand, for E not much larger than S. In inert gases, at pressures ranging from a few mm. Hg to 3 or 4 cm. Hg, with the electrodes a few mm. apart, the discharge starts at from 100 to 300 volts and the difference $S - O$ has a value of 20 to 50 volts. The intermittences are best obtained with tubes in which the anode is slightly smaller than the cathode, the anode being for instance a plate of 12 to 15 sq. cm., surrounded by the U-shaped iron cathode, or a small anode facing a cathode-wire wound in a spiral. E being chosen as 200 volts, the values of $ln \frac{E - O}{E - S}$ are usually not much larger than unity

so that very roughly $t' = Cr$. The high resistance "r" required in the circuit may be a vacuum tube with 2 electrodes; the resistance which it represents being variable between wide limits if the heating current of the filament is changed. When operated at saturation the charging current remains constant; the charging up rate is therefore a linear function of the time

$$dV/dt = i/C \qquad \text{or} \qquad t' = \frac{C}{i_s}(S - O).$$

The same method furnishes a linear time basis for cathode ray oscillographs.

It is possible to get interruptions with the capacities associated with the electrodes (60 cm.), or at low pressures, with the condenser formed by the electrodes and the glass wall. Still higher frequencies are produced when the discharge tube is placed in a transverse magnetic field of a few 1000 gauss.

The interrupted currents are useful for generating alternating currents in a transformer, or for producing audible frequencies in a telephone receiver, or loud speaker; they may be amplified by means of a vacuum tube circuit in parallel with the condenser, the grid being maintained below the potential of the cathode with the help of an auxiliary battery. In such cases it is advantageous to have "t'" as short as possible, and to have a large difference between S and O. "S" is known to pass through a smallest value at a few mm. pressure, the exact value depending upon the shape of the electrodes; moreover S increases with pressure. Fairly high pressures are best for producing strong alternations.

The intermittences in a glow tube circuit are very rich in higher harmonics, since they consist of current impulses falling off rapidly and followed by dead intervals of long duration. The higher harmonics are strong enough to excite tuned oscillating circuits, and to produce beats with incoming radio frequencies; this is shown by the following example:

Interruptions per sec. (fundamental).....	10 to 100.	100 to 1000.	1000 to 3000.
First harmonic.....	5%	10%	20%
Second "	5	10	16
Third "	5	10	13
Fourth "	5	9	10

When the system to be set in motion by the interrupted current possesses itself a marked natural frequency of oscillation, it may act back upon the exciting circuit; thus the prongs of a tuning fork, or a steel diaphragm, actuated by an electromagnet induce an alternating voltage $V_0 \cos(pt + \varphi)$ in the primary circuit. This induced electromotive force, even if of only a few volts, may alter the period of the discharge and render the discharge more uniform and more regular. With proper values of L and C, the frequency of the flashes becomes equal to the frequency of the vibrating mechanical system, the condition to be satisfied being

$$V_0^2 \text{ proportional to } (-\cos\varphi/Cr - \sin\varphi) > 0.$$

The frequencies obtained in an interrupted discharge vary from one discharge per hour to a few thousand discharges per sec. Beyond this point, recombination is not fast enough to remove the ions from the gas, space charges remain, and by reducing the starting voltage, decrease the energy of the intermittent current. At the limit, a continuous discharge results. The frequencies obtainable embrace therefore the whole range of sound waves.

The intermittent discharge giving audible frequencies may be used for determining rapidly large capacities and resistances, and for detecting and measuring leakage of condensers and cables; because when the circuit is connected with a telephone receiver, the pitch of the sound produced depends on C and r. The method is applicable to values of C varying from 10 to 10^7 cm. and of r varying from 10^3 to 10^8 ohms. Condensers offering a resistance of less than 100 megohms are easily detected. Measurements are made by substituting in the circuit a known resistance, or a known capacity for the unknown, until a constant pitch given by a tuning fork or by a beat frequency, is reproduced. An accuracy of 1 per cent. is not difficult to obtain. The method can also be applied to the determination of dielectric constants, or to the measurement of the concentration of dielectric materials, for instance the amount of moisture, in a given sample of material.

GAS-FILLED PHOTO-ELECTRIC CELLS.[1]

The self-interrupting discharge is a useful tool for the study of photo-electric currents. The stream of electrons liberated from a metal surface in vacuum under the influence of light is comparatively small, so that delicate and expensive instruments are necessary for its measurement. By introducing gas, and by increasing at the same time the potential applied to the anode, to a value which causes ions to be formed, the number of charges being collected is multiplied many hundred and thousand times. The potential must, of course, remain below the value at which the unsupported glow discharge sets in, unless the circuit is so arranged that the discharge is self-interrupting. If (in Fig. 7) the discharge tube is replaced by a gas-filled photo-electric cell (Fig. 8), exposed to visible radiant energy, the charging up

FIG. 8.

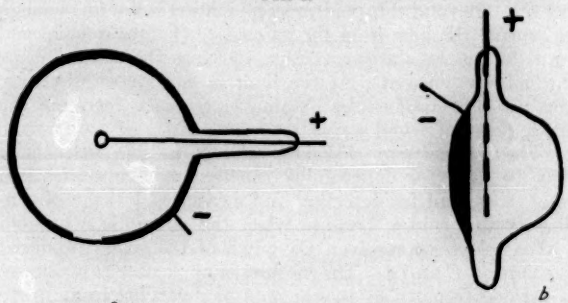


Photo-electric cells with (a) spherical and (b) plane active surface.

of the condenser will be retarded in consequence of the electron current flowing across the photo-electric cell. The stronger therefore the illumination, the slower the frequency of the discharge pulses. With spherical cells this method is suitable for the detection of very small illuminations, as for instance the illumination produced by one of the brighter stars, without requiring elaborate apparatus, a telephone inserted in one of the battery leads being sufficient. The range over

[1] N. R. Campbell, *Phil. Mag.*, 4, p. 726 (1927).
Campbell and Ritchie, Photo-electric Cells 1930 (216 p.).

which intermittences can be obtained with a given cell is of course limited, to a certain value of illumination; when the illumination is too strong, the Townsend current passes without any discontinuity into a steady discharge, which is not altered by cutting off the light. When the illumination is too small, a sudden increase of the current takes place when the starting potential is reached. As approximately the same quantity of electricity CV is discharged at each flash, the average current is a function, and a measure of the illumination to which the cell is exposed. When using spherical cells, it is best to set the cell so as to give few flashes, say 2 per sec., in the dark; with plane cells, in which surface charges are likely to accumulate, the current is adjusted just below that at which the oscillations cease, and the steady discharge sets in (N. R. Campbell); the light is found to stop the high audio oscillations and to cause the steady current to flow.

Gas filled cells are widely used in talking films, where the intensity of the light transmitted by the edge of the film varies at speech frequency, and gives rise, after suitable amplification, to speech currents in the loud-speaker. On account of their high sensitivity, gas-filled cells are also used in protection devices, against fire and smoke, for instance, or for operating warning signals when trains approach a level-crossing, or for detecting flaws and imperfections in woven, or colored, material. On account of their quick response, they are of great help for the accurate timing of events, as during races, for counting and controlling the speed of rotating wheels, or for regulating the flow of liquids. In most of these cases it is advantageous to expose the cell to the light from an incandescent lamp, which is obscured "n" times per second ("n" from 20 to 2000) by means of a rotating sector; the "n" current impulses, representing an audio frequency current can be amplified, quite independent of the steady, or slowly changing, day-light. For the comparison, or matching, of colors, or of light intensities, two photo-cells are placed in two of the arms of a Wheatstone bridge; whenever the cells receive unequal amounts of light, the needle of the indicating instrument of the needle is deflected.

Gaseous photo-electric cells are on the market under different trade-names.

I. OLDER TYPES.

	Anode.	Cathode.	Gas.	Light Window (cm.).	Sensit. 10^{-6} A per foot-cdle.
Cambridge cell...	central loop	KH upon Ag KH upon glass wall	He	diam. = 2.5	
El. Eye.....	Ni-plate	KH	Ne 3 mm.	diam. = 4.3	
KH tube.....	2.5 cm. loop	KH upon Mg KH upon glass wall	Ne 3 mm.	diam. = 2.5	
Phototron.....	4 crossed wires 2.5×3 cm.	Na or K on central Ni-plate	Ne 3 mm.	1×3	
Visitron.....	hexagonal loop 2.5×3	KH upon Ag KH upon glass wall	A	2.5×2.5	
Raytheon 3GS...	central loop	"		diam. = 5	0.15
Raytheon 3GL..	"	"			0.35
Raytheon 3GLL.	"	"			0.75

II. RECENT TYPES.

(*a*) Blue sensitive cells (maximum near 4300A corresponding to a black body temperature of 7000K.)

	Operating Voltage.	Sparking Potential Volts at "*l*" Lumen.	Appr. Sensitive Surface sq. cm.	Lumens per foot-cdle.	Photoconductance 10^{-6}A per Lumen $= \Delta i/\Delta l$.	Conductance $\Delta i/\Delta e$ at *l* Lumens.
Raytheon 3GC.....	135	150 at 0.45	3×3		10×10^{-6}	10^{-7}
" 3GS.....	135	150 " 1.4	5×5	0.02	8 "	3×10^{-7}
" 3GP.....	275	300 " 0.8	3.5×3.5		10 "	2.5×10^{-7}
" 3GL.....	135	150 " 6.0	3.5×2.1	0.08	4.5 "	8×10^{-7}
" 3GSM.....	135	150 " 0.35	1.5×3		10 "	10^{-7}
Jenkins SR1–16 etc...	135	150 " 0.35	3.7×3.7		30 "	10^{-6}
" SR2–16 " ..	"	"	"			
" SR2–80 " ..			24×24			

(*b*) Red sensitive cells (secondary maximum near 6600 to 7600A.).

Raytheon 4GSM.....	90	100 at 0.7	3×3.5		20	10^{-6}
4GSM.....	90	100 " 0.4	1.5×3.5		20	3×10^{-7}
Visitron 53A.....	90	90 " 0.2	2.5×1.2	0.003	40	8×10^{-7}
58A.....	90	90 " 0.2	2.5×1.5	0.004	40	
71A.....	90	90 " 0.2	2.5×3	0.008	40	
75A.....	90	90 " 0.2	2.7×4	0.013	40	

THE DISCHARGE TUBE AS A LIGHT SOURCE.[10]

Gases at low pressures, which are traversed by the electric current, emit a more or less abundant number of spectral lines, the strongest of which may be isolated by means of filters and prisms, and constitute the present day sources of monochromatic light (Abbe; Michelson). These radiations are used in the scientific as well as in the rapidly expanding industrial work with interferometers. Heavy gases, at low temperatures, such as Kr, Xe, give fine lines for high precision work, the He tube and the Hg arc are convenient sources for routine examinations. Discharge tubes constitute strong sources of ultra-violet radiation.

The characteristic color of electrically excited gases makes the discharge a very conspicuous source of light. As a result of the work of G. Claude, a.o., the first Ne commercial signs made their appearance in France in 1914, and mark to-day many thoroughfares and buildings of the larger cities. Owing to their decided red orange color, sunshine has little effect on their brilliancy, so that they are effective day and night. Although Neon signs cost slightly more than the other types of electric signs, the cost of current for the usual dimensions of letters and figures is only about one-half of that of incandescent lamps. A tube runs on the average satisfactorily for a whole year, day and night. The red waves of the Ne glow traverse foggy air for a considerable distance (beacons for air routes in France, England and the United States).

As long as the electrodes are not completely covered by the discharge, the area of the negative glow is proportional to the current. The light emitted by a discharge without positive column is therefore proportional to the current. The discharge tube thus constitutes the reverse of a photo-electric cell. If the electrodes be two long wires separated by a short gap, the length of the discharge glow will be proportional to the current, and in the case of alternating current the variations in the length of the glow are a true picture of the current variations as long as the voltage remains above the normal cathode fall, or the operating potential of the

[10] R. Seeliger and M. Reger, *Annalen d. Physik.*, **83**, p. 535 (1927); **86**, p. 1089 (1928). M. Alterthum, M. Reger and R. Seeliger, *Zeitschr. techn. Physik.*, **9**, p. 161 (1928).

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M. Reger, *Annalen d. Physik.*, 83, p. 535 (1927); 86, p. 1089
n, M. Reger and R. Seeliger, *Zeitschr. techn. Physik.*, 9,

44

discharge. The voltage drop across the discharge space is constant despite the changes in current, and can be compensated for by an auxiliary source. As the last stages of the audio amplifiers in modern radio receivers, or loud speakers, are operated at several hundred volts, the audio-frequency current may run a discharge tube, the light from the tube may be photographed upon a film so that a permanent record is obtained. When now light from a constant source is thrown upon the film, and the transmitted light is made to act on a photo-electric cell, the voice currents will be reproduced, as far as the blackening of the film is proportional to the incident light of the discharge glow (Talkies). Throughout the range of audio-frequencies there is no lag between the voltage and the luminosity, a property which makes the Ne tube valuable for stroboscopic work.

DYNAMICAL ANALYSIS OF MACHINES.*

BY

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Member of the Institute.

ON THE ROLLING OF CYLINDERS IN CONTACT (Fig. 73)

As an example of a rolling system with two degrees of freedom, we will consider the problem of an inner cylinder of mass m, and radius of gyration k, which rolls on the inner circumference of a large outer cylinder which latter, in turn, is free to roll on a rough horizontal plane.

The kinematics of this mechanism are related as follows: The condition for the common motion at the contact of the cylinders is, if "b" is the radius of the outer cylinder and "a" that of the inner:

$$b\phi = (b - a)\theta - \psi a;$$

$$\therefore \quad \psi = \frac{b - a}{a}\theta - \frac{b}{a}\phi.$$

The system has, therefore, two degrees of freedom defined by θ and ϕ.

The equation of energy is obtained from the component energies:

$$\tfrac{1}{2}Mb^2\dot{\phi}^2 + \tfrac{1}{2}MK^2\dot{\phi}^2 = \tfrac{1}{2}I\dot{\phi}^2$$

for the outer cylinder,

$$\tfrac{1}{2}m[(b - a)^2\dot{\theta}^2 + b^2\dot{\phi}^2 - 2(b - a)b\dot{\theta}\dot{\phi}\cos\theta] + \tfrac{1}{2}mk^2\dot{\psi}^2$$

for the inner cylinder.

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* Extension of a portion of a dissertation for the degree of doctor of philosophy submitted to Clark University, 1928.

Continued from p. 503, vol. 210, October, 1930.

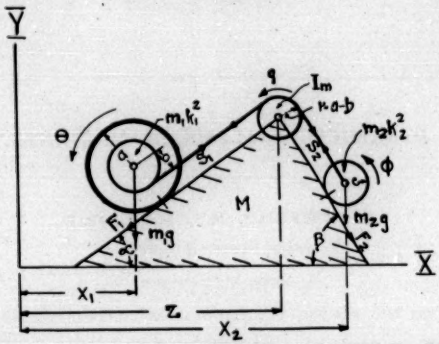

fig. 72

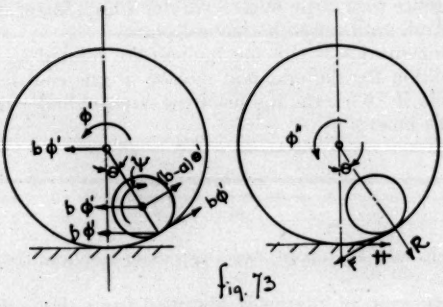

fig. 73

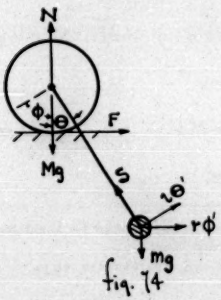

fig. 74

$$\therefore \quad T = \frac{1}{2}\left(I + mb^2 + m\frac{b^2}{a^2}k^2\right)\dot{\phi}^2 + \frac{1}{2}m\left((b-a)^2 + k^2\frac{(b-a)^2}{a^2}\right)\dot{\theta}^2 - m\left((b-a)b\cos\theta + \left(\frac{b-a}{a^2}\right)bk^2\right)\dot{\theta}\dot{\phi},$$

$$\frac{\partial T}{\partial \dot{\phi}} = \left(I + mb^2 + \frac{mb^2}{a^2}k^2\right)\dot{\phi} - m\left((b-a)b\cos\theta + \left(\frac{b-a}{a^2}\right)bk^2\right)\dot{\theta}$$

and $\dfrac{\partial T}{\partial \phi} = 0,$

$$\frac{\partial T}{\partial \dot{\theta}} = m\left((b-a)^2 + k^2\frac{(b-a)^2}{a^2}\right)\dot{\theta} - m\left((b-a)b\cos\theta + \frac{(b-a)}{a^2}bk^2\right)\dot{\phi},$$

$$\frac{\partial T}{\partial \theta} = m(b-a)b\sin\theta\cdot\dot{\theta}\dot{\phi}.$$

Also,

$$V = -mg(b-a)\cos\theta + \text{constant};$$

$$\therefore \quad \frac{\partial V}{\partial \theta} = mg(b-a)\sin\theta, \qquad \frac{\partial V}{\partial \phi} = 0.$$

Therefore, the equations of motion are

$$\left(I + mb^2 + m\frac{b^2}{a^2}k^2\right)\ddot{\phi} - m\left((b-a)b\cos\theta + \left(\frac{b-a}{a^2}\right)bk^2\right)\ddot{\theta} + m(b-a)b\sin\theta\cdot\dot{\theta}^2 = 0 \quad (1)$$

and

$$m\left((b-a)^2 + \frac{(b-a)^2}{a^2}k^2\right)\ddot{\theta} - m\left((b-a)b\cos\theta + \left(\frac{b-a}{a^2}\right)bk^2\right)\ddot{\phi} + mg(b-a)\sin\theta = 0. \quad (2)$$

The equations may also be obtained by the method of velocity ratios. The condition equation for the rolling of the inner cylinder on the outer is

$$\delta\psi = \frac{b-a}{a}\delta\theta - \frac{b}{a}\delta\phi,$$

so that

$$K_{\psi\theta} = \frac{b-a}{a} \quad \text{and} \quad K_{\psi\theta} = -\frac{b}{a},$$

which evidently implies first holding the outer cylinder fixed, i.e., ϕ constant, and then maintaining the radius vector of the inner cylinder with respect to the vertical while moving the outer cylinder, constant, i.e., θ is kept constant.

For the coördinate displacements (absolute) of the axis of the inner cylinder, we have

$$\delta x = (b-a)\delta\theta\cos\theta - b\delta\phi,$$
$$\delta y = (b-a)\delta\theta\sin\theta,$$

$$K_{x\theta} = (b-a)\cos\theta, \qquad K_{y\theta} = (b-a)\sin\theta, \qquad K_{x\phi} = -b,$$

$$\frac{\partial K_{x\theta}}{\partial\theta} = -(b-a)\sin\theta, \quad \frac{\partial K_{y\theta}}{\partial\theta} = (b-a)\cos\theta,$$

which are the partial velocity ratios and their derivatives for the inner cylinder of mass m.

For the outer cylinder, evidently, $K_{x_1\phi} = -b$.

For the work of the applied forces we have

$$-mg\delta y = -mg\frac{\partial y}{\partial\theta}\delta\theta = -mgK_{y\theta}\delta\theta,$$

so that the applied force is $-mgK_{y\theta}$.

The equations of motion in terms of the partial velocity ratios and their derivatives are

$$mk^2\left[\left(K_{x\theta}^2 + K_{y\theta}^2\right)\ddot{\theta} + K_{x\theta}K_{x\phi}\ddot{\phi} + \left(K_{x\theta}\frac{\partial K_{x\theta}}{\partial\theta} + K_{y\theta}\frac{\partial K_{y\theta}}{\partial\theta}\right)\dot{\theta}^2\right]$$
$$+ mk^2\left[K_{\psi\theta}^2\ddot{\theta} + K_{\psi\theta}K_{\psi\phi}\ddot{\phi}\right] = -mgK_{y\theta}$$

and

$$m\left[(K_{x\phi}{}^2\ddot{\phi} + K_{x\phi}K_{x\theta}\ddot{\theta}) + K_{x\phi}\frac{\partial K_{x\theta}}{\partial\theta}\cdot\dot{\theta}^2 \right]$$
$$+ mk^2(K_{x\phi}{}^2\ddot{\phi} + K_{\psi\phi}K_{\psi\theta}\ddot{\theta}) + MK_{x_1\phi}^2\ddot{\phi} + MK^2\ddot{\phi} = 0.$$

Substituting for the partial velocity ratios, we obtain

$$m[(b-a)^2\ddot{\theta} - b(b-a)\cos\theta\cdot\ddot{\phi}]$$
$$+ mk^2\left[\frac{(b-a)^2}{a^2}\ddot{\theta} - \frac{b(b-a)}{a^2}\ddot{\phi}\right] + mg(b-a)\sin\theta = 0$$

and

$$m[b^2\ddot{\phi} - b(b-a)\cos\theta\cdot\ddot{\theta} + b(b-a)\sin\theta\cdot\dot{\theta}^2]$$
$$+ mk^2\left[\frac{b^2}{a^2}\ddot{\phi} - \frac{b(b-a)}{a^2}\ddot{\theta}\right] + M(b^2+K^2)\ddot{\phi} = 0,$$

where $M(b^2+K^2) = I$, which verifies equations (2) and (1) by the Lagrangian method.

These equations may be verified directly by combining the equations of motion, taking, of course, into consideration the kinematic relations.

In Fig. 73 let:

F = mutual frictional component between the two cylinders at their point of contact.
R = the corresponding normal component.
H = the tangential frictional component of the reaction of the plane on the outer cylinder.
a = radius of inner cylinder.
b = radius of outer cylinder.

Then using the same angular coördinates the equations of motion are

$$F - mg\sin\theta = m((b-a)\ddot{\theta} - b\ddot{\phi}\cos\theta), \quad (1)$$
$$R - mg\cos\theta = m((b-a)\dot{\theta}^2 + b\ddot{\phi}\sin\theta), \quad (2)$$
$$-Fa = mk^2\ddot{\phi}, \quad (3)$$
$$F\cos\theta - R\sin\theta - H = Mb\ddot{\phi}, \quad (4)$$
$$-Fb + Hb = MK^2\ddot{\phi} \quad (5)$$

and

$$\psi = \frac{b-a}{a}\theta - \frac{b}{a}\phi.$$

Combining (4) and (5),

$$Fb(\cos\theta - 1) - Rb\sin\theta = I\ddot{\phi}. \tag{6}$$

Substituting for F and R,

$$b[mg\sin\theta + m((b-a)\ddot{\theta} - b\ddot{\phi}\cos\theta))(\cos\theta - 1) - (mg\cos\theta + m((b-a)\dot{\theta}^2 + b\dot{\phi}\sin\theta))b\sin\theta = I\ddot{\phi}. \tag{7}$$

From eqs. (1) and (3),

$$-\frac{mk^2\ddot{\psi}}{a} - mg\sin\theta = m((b-a)\ddot{\theta} - b\ddot{\phi}\cos\theta);$$

$$\therefore\ -\frac{mk^2}{a}\left(\left(\frac{b-a}{a}\right)\ddot{\theta} - \frac{b}{a}\ddot{\phi}\right) - mg\sin\theta = m(b-a)\ddot{\theta} - mb\cos\theta\cdot\ddot{\phi}. \tag{8}$$

This equation is identical to eq. (2) by the Lagrangian method.

Combining eqs. (7) and (8), that is, substituting the value of $mgb\sin\theta$ of (7) in eq. (8), we obtain

$$\left(I + mb^2 + \frac{b^2}{a^2}k^2\right)\ddot{\phi} - m\left(b(b-a)\cos\theta + \left(\frac{b-a}{a^2}\right)bk^2\right)\ddot{\theta} + m(b-a)b\dot{\theta}^2\sin\theta = 0,$$

which verifies eq. (1) by the Lagrangian method.

PENDULUM SUSPENDED FROM THE AXIS OF A ROTATING CYLINDER WHICH ROLLS ON A HORIZONTAL PLANE (Fig. 74).

Let the mass m of the pendulum be connected by a light rod to the axis of the cylinder of mass M and moment of inertia about its center of gravity I. As the cylinder turns independently from the oscillation of the pendulum the system has two degrees of freedom. The angular coördinate of the cylinder is ϕ and that of the pendulum and rod θ. Let l be the length of the pendulum rod with y axis vertically down and the x axis along the plane.

The work function and potential are related by

$$\delta W = mg\delta y = -\delta V$$

or

$$\delta W = mg\left(\frac{\partial y}{\partial \theta}\delta\theta + \frac{\partial y}{\partial \phi}\delta\phi\right) = -\delta V;$$

$$\therefore \quad \Phi_\theta = mg\frac{\partial y}{\partial \theta} \quad \text{and} \quad \Phi_\phi = mg\frac{\partial y}{\partial \phi};$$

but

$$y = l\cos\theta - r, \qquad \therefore \quad \frac{\partial y}{\partial \theta} = -l\sin\theta, \qquad \frac{\partial y}{\partial \phi} = 0,$$

$$\Phi_\theta = -mgl\sin\theta, \qquad \text{and} \qquad \Phi_\phi = 0.$$

The kinetic energy is

$$T = \tfrac{1}{2}(I\dot\phi^2 + Mr^2\dot\phi^2) + \tfrac{1}{2}m(l^2\dot\theta^2 + r^2\dot\phi^2 + 2lr\dot\theta\dot\phi\cos\theta);$$

$$\therefore \quad \frac{\partial T}{\partial \dot\theta} = m(l^2\dot\theta + lr\dot\phi\cos\theta), \qquad \frac{\partial T}{\partial \theta} = -mlr\sin\theta\dot\theta\dot\phi,$$

$$\frac{\partial T}{\partial \dot\phi} = (I + mr^2)\dot\phi + m(r^2\dot\phi + lr\cos\theta\cdot\dot\theta), \qquad \frac{\partial T}{\partial \phi} = 0.$$

The equations of motion are, therefore,

$$ml^2\ddot\theta + mlr\cos\theta\ddot\phi = -mgl\sin\theta,$$

$$(I + (M + m)r^2)\ddot\phi + mlr\cos\theta\cdot\ddot\theta - mlr\sin\theta\cdot\dot\theta^2 = 0.$$

By the Direct Method:

Let S = tension in the rod,

F = tangential frictional component of the reaction of the frame on the roller.

Then, for the translation and rotation of the roller alone,

$$S\sin\theta + F = Mr\ddot\phi, \tag{1}$$

$$-Fr = I\ddot\phi \qquad \text{or} \qquad F = -\frac{I}{r}\phi; \tag{2}$$

$$\therefore \quad S\sin\theta = Mr\ddot\phi + \frac{I}{r}\ddot\phi = \left(\frac{Mr^2 + I}{r}\right)\ddot\phi, \tag{3}$$

and for the motion of the bob,

$$S - mg\cos\theta = ml\dot{\theta}^2 - mr\ddot{\phi}\sin\theta, \tag{4}$$

$$-mg\sin\theta = mr\ddot{\phi}\cos\theta + ml\ddot{\theta}. \tag{5}$$

Multiplying (4) by $\sin\theta$ and then substituting the value of S in (3), we obtain

$$mgr\sin\theta\cos\theta = ((M+m)r^2 + I)\ddot{\phi} - mlr\dot{\theta}^2\sin\theta - mr^2\ddot{\phi}\cos^2\theta. \tag{6}$$

Multiplying (5) by $r\cos\theta$ and adding to (6), we obtain

$$(I + (M+m)r^2)\ddot{\phi} + mlr\cos\theta\cdot\ddot{\theta} - mlr\sin\theta\cdot\dot{\theta}^2 = 0. \tag{7}$$

Thus, eqs. (5) and (7) verify eqs. (1) and (2) obtained by the Lagrangian method.

We will now consider the derivation of the equations by the method of velocity ratios.

Since the coördinates of "m" are related by the condition equations,

$$\delta x = r\delta\phi + l\delta\theta\cos\theta, \qquad \delta y = l\delta\theta\sin\theta.$$

The velocity ratios of "m" are

$$K_{x\phi} = r, \qquad K_{x\theta} = l\cos\theta, \qquad \frac{\partial K_{x\theta}}{\partial\theta} = -l\sin\theta,$$

$$K_{y\phi} = 0, \qquad K_{y\theta} = l\sin\theta, \qquad \frac{\partial K_{y\theta}}{\partial\theta} = l\cos\theta,$$

which could be obtained by first holding ϕ constant and then θ constant. The velocity ratios for M are obtained from the relation

$$\delta x_1 = r\delta\phi; \qquad \therefore \quad K_{x_1\phi} = r.$$

The equations of motion are, therefore,

$$m\left[(K_{x\theta}^2 + K_{y\theta}^2)\ddot{\theta} + K_{x\theta}K_{x\phi}\ddot{\phi} + \left(K_{x\theta}\frac{\partial K_{x\theta}}{\partial\theta} + K_{y\theta}\frac{\partial K_{y\theta}}{\partial\theta}\right)\dot{\theta}^2\right] = mgK_{y\theta},$$

$$m\left[K_{x\phi}^2\ddot{\phi} + K_{x\phi}K_{x\theta}\ddot{\theta} + K_{x\phi}\frac{\partial K_{x\theta}}{\partial\theta}\dot{\theta}^2\right] + (I + MK_{x_1\phi})\ddot{\phi} = 0.$$

On substituting for the velocity ratios we verify the previous equations of motion.

To obtain the normal and tangential components of the reaction of the horizontal plane we will apply the generalized equations of momentum in terms of velocity ratios. Simplifying, we obtain

$$N = m\left(K_{y\theta}\ddot{\theta} + \frac{\partial K_{y\theta}}{\partial\theta}\cdot\dot{\theta}^2\right) + (M+m)g,$$

$$F = m\left(K_{x\theta}\ddot{\theta} + K_{x\phi}\ddot{\phi} + \frac{\partial K_{x\theta}}{\partial\theta}\dot{\theta}^2\right) + MK_{x_1\phi}\ddot{\phi}.$$

Substituting for the velocity ratios, we have

$$N = m(l\sin\theta\cdot\ddot{\theta} + l\cos\theta\cdot\dot{\theta}^2) + (M+m)g,$$
$$F = m(l\cos\theta\cdot\ddot{\theta} + r\ddot{\phi} - l\sin\theta\cdot\dot{\theta}^2) + Mr\ddot{\phi}.$$

MOTION OF A WHEEL ROLLING ON A MOVING CYLINDRICAL WEDGE (Fig. 75).

We will assume a wheel of mass m and radius of gyration k to roll on the rough surface of a cylindrical wedge of mass M, the latter sliding in turn on a frictionless horizontal plane. Let a = the radius of the wheel, and b = the radius of the cylindrical curvature of the wedge.

A force P is applied horizontally to the wedge. The coördinates of the system are two, the displacement of the wedge z and the relative displacement of the wheel along the circular groove of the wedge. The latter can be expressed by the angle θ of the radius vector through the center of the wheel with respect to the vertical. The angular coördinate of the wheel itself ϕ is not independent but is connected by the condition relation

$$(b-a)\delta\theta - a\delta\phi = 0; \qquad \therefore \quad \delta\phi = \frac{b-a}{a}\delta\theta$$

and

$$K_{\phi\theta} = \frac{b-a}{a}.$$

For the coördinates of the wheel center, we have

$$\delta x = \delta z + (b-a)\delta\theta\cos\theta,$$
$$\delta y = (b-a)\delta\theta\sin\theta,$$

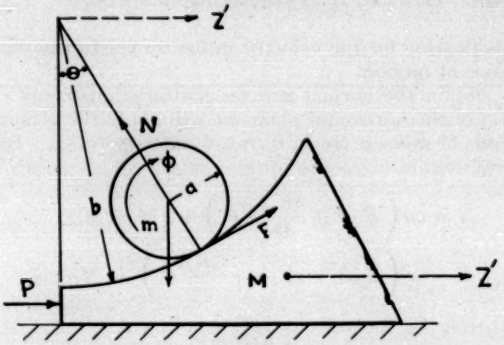

fig. 75

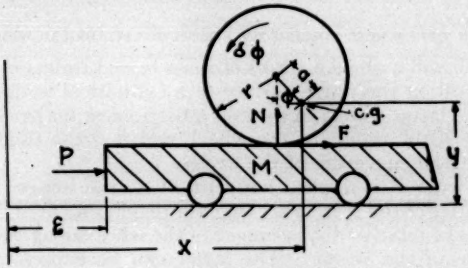

fig. 75a

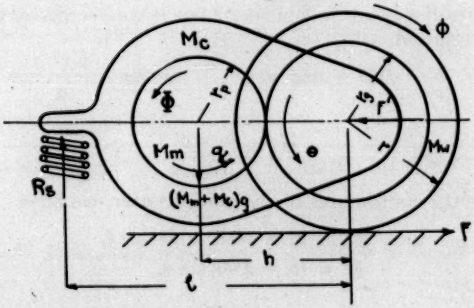

fig. 76

$$K_{xz} = 1, \qquad K_{x\theta} = (b-a)\cos\theta, \qquad \frac{\partial K_{x\theta}}{\partial\theta} = -(b-a)\sin\theta,$$

$$K_{yz} = 0, \qquad K_{y\theta} = (b-a)\sin\theta, \qquad \frac{\partial K_{y\theta}}{\partial\theta} = (b-a)\cos\theta.$$

The work of the applied forces are

$$P\delta z - mg\delta y = P\delta z - mg(b-a)\sin\theta\cdot\delta\theta,$$

so that the applied forces corresponding to z and θ are P and

$$-mg(b-a)\sin\theta$$

The equations of motion are

$$M\ddot{z} + m\left(K_{xz}^2\ddot{z} + K_{xz}K_{x\theta}\ddot{\theta} + K_{xz}\frac{\partial K_{x\theta}}{\partial\theta}\dot{\theta}^2\right) = P,$$

$$m\left((K_{x\theta}^2 + K_{y\theta}^2)\ddot{\theta} + K_{x\theta}K_{xz}\ddot{z} + \left(K_{x\theta}\frac{\partial K_{x\theta}}{\partial\theta} + K_{y\theta}\frac{\partial K_{y\theta}}{\partial\theta}\right)\dot{\theta}^2\right.$$
$$\left. + mk^2K_{\phi\theta}^2\ddot{\theta} = -mg(b-a)\sin\theta.\right.$$

On substituting for the velocity ratios we obtain

$$(M+m)\ddot{z} + m((b-a)\cos\theta\cdot\ddot{\theta} - (b-a)\sin\theta\cdot\dot{\theta}^2) = P,$$

$$m\left((b-a)^2\ddot{\theta} + (b-a)\cos\theta\cdot\ddot{z}\right) + mk^2\left(\frac{b-a}{a}\right)^2\ddot{\theta}$$
$$= -mg(b-a)\sin\theta.$$

By the Lagrangian method, since,

$$T = \frac{1}{2}\left[M\dot{z}^2 + m((b-a)\dot{\theta}^2 + \dot{z}^2 + 2(b-a)\dot{\theta}\dot{z}\cos\theta)\right.$$
$$\left. + mk^2\left(\frac{b-a}{a}\right)^2\dot{\theta}^2\right].$$

Therefore,

$$\frac{\partial T}{\partial\dot{z}} = M\dot{z} + m\dot{z} + m(b-a)\dot{\theta}\cos\theta, \qquad \frac{\partial T}{\partial z} = 0;$$

$$\therefore \quad P = (M+m)\ddot{z} + m[(b-a)\ddot{\theta}\cos\theta - (b-a)\sin\theta\cdot\dot{\theta}^2].$$

If F and N are the tangential and normal components between the wheel and wedge, we have, also

$$M\ddot{z} = P - F\cos\theta + N\sin\theta, \qquad \text{for wedge,}$$

$$\left.\begin{aligned} m(b-a)\ddot{\theta} + m\ddot{z}\cos\theta &= F - mg\sin\theta, \\ m(b-a)\dot{\theta}^2 - m\ddot{z}\sin\theta &= N - mg\cos\theta, \end{aligned}\right| \quad \text{for wheel.}$$

On eliminating the reactions we obtain the previous results.

Again,

$$\frac{\partial T}{\partial \dot{\theta}} = m(b-a)^2\dot{\theta} + m(b-a)\dot{z}\cos\theta + mk^2\left(\frac{b-a}{a}\right)^2\dot{\theta}$$

and

$$\frac{\partial T}{\partial \theta} = -m(b-a)\dot{\theta}\dot{z}\sin\theta.$$

Hence,

$$m[(b-a)^2\ddot{\theta} + (b-a)\ddot{z}\cos\theta] + mk^2\left(\frac{b-a}{a}\right)^2\ddot{\theta} = -mg(b-a)\sin\theta,$$

thus agreeing with the result by velocity ratios. We note

$$m[(b-a)\ddot{\theta} + \ddot{z}\cos\theta] = F - mg\sin\theta,$$

where

$$F = \frac{mk^2\ddot{\phi}}{a} = mk^2\left(\frac{b-a}{a^2}\right)\ddot{\theta}.$$

On eliminating F we obtain the above expression.

The magnitude of P could also have been obtained from the momentum equation

$$M\ddot{z} + m\left[K_{zz}\ddot{z} + K_{z\theta}\ddot{\theta} + \frac{\partial K_{z\theta}}{\partial\theta}\cdot\dot{\theta}^2\right] = P$$

$$\therefore \quad (M+m)\ddot{z} + m((b-a)\cos\theta\cdot\ddot{\theta} - (b-a)\sin\theta\cdot\dot{\theta}) = P.$$

REACTION OF AN ECCENTRIC WEIGHTED ROLLER ON AN ACCELERATEDFRAME.

This problem offers a very simple solution by the method of velocity ratios. The system has two degrees of freedom, ξ for the translation of the frame of mass M and ϕ for the

angle turned by the roller. Let a be the eccentricity of the roller from its axis and r its outer radius.

The coördinates of the c. of g. of the roller x and y are related to the coördinates ξ and ϕ by the condition relations

$$\delta x = \delta\xi - (r - a\cos\phi)\delta\phi, \qquad \delta y = a\delta\phi\sin\phi,$$

so that the velocity ratios are

$$K_{x\xi} = 1, \qquad K_{x\phi} = -(r - a\cos\phi), \qquad \frac{\partial K_{x\phi}}{\partial\phi} = -a\sin\phi,$$

$$K_{y\xi} = 0, \qquad K_{y\phi} = a\sin\phi, \qquad \frac{\partial K_{y\phi}}{\partial\phi} = a\cos\phi.$$

The forces acting on the system which do work are the translation force P acting on the frame and the weight of the roller acting at its c. of g.

Since

$$P\delta\xi - mg\delta y = P\delta\xi - mga\delta\phi\sin\phi,$$

the forces corresponding to the coördinates ξ and ϕ are

$$P \text{ and } -mga\sin\phi.$$

The equations of motion are

$$m\left(K_{x\xi}^2\ddot{\xi} + K_{x\xi}K_{x\xi}\ddot{\phi} + K_{x\xi}\frac{\partial K_{x\phi}}{\partial\phi}\cdot\dot{\phi}^2\right) + M\ddot{\xi} = P, \quad (1)$$

$$m\left((K_{x\phi}^2 + K_{y\phi}^2)\ddot{\phi} + K_{x\phi}K_{x\xi}\ddot{\xi} + \left(K_{x\phi}\frac{\partial K_{x\phi}}{\partial\phi} + K_{y\phi}\frac{\partial K_{y\phi}}{\partial\phi}\right)\dot{\phi}^2\right) + mk^2\ddot{\phi} = -mga\sin\phi. \quad (2)$$

On substituting for the velocity ratios we have

$$m(\ddot{\xi} - (r - a\cos\phi)\ddot{\phi} - a\sin\phi\cdot\dot{\phi}^2) + M\ddot{\xi} = P, \quad (1)$$

$$m(k^2 + r^2 + a^2 - 2ra\cos\phi)\ddot{\phi} - m(r - a\cos\phi)\ddot{\xi} + mra\sin\phi\cdot\dot{\phi}^2 = -mga\sin\phi. \quad (2)$$

Since $K_{x\xi} = 1$, we note eq. (1) becomes, as we should expect, identical to the equation of momentum for the translation of the system.

To determine the mutual tangential and normal component reactions F and N between the frame and roller, we have

$$F = m\left(K_{x\xi}\ddot{\xi} + K_{x\phi}\ddot{\phi} + \frac{\partial K_{x\phi}}{\partial \phi} \cdot \dot{\phi}^2\right),$$

$$N = m\left(K_{y\phi}\ddot{\phi} + \frac{\partial K_{y\phi}}{\partial \phi} \cdot \dot{\phi}^2\right) + mg;$$

hence,

$$F = m(\ddot{\xi} - (r - a\cos\phi)\ddot{\phi} - a\sin\phi\cdot\dot{\phi}^2),$$

$$N = m(a\sin\phi\cdot\ddot{\phi} + a\cos\phi\cdot\dot{\phi}^2) + mg.$$

(To be continued.)

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

AERONAUTIC RADIO RESEARCH—THE "DEVIOMETER."

One of the auxiliary devices which the research division of the Aeronautics Branch, Department of Commerce has developed to facilitate the use of the visual type range beacon is an instrument called a "deviometer." By its use a pilot can follow any chosen course, within limits, on either side of the equisignal line for which the beacon transmitter is adjusted. Recent developments indicate that this device will be of considerable practical value. The following papers prepared by the research division give information on this instrument.

"Radio developments applied to aircraft." J. H. Dellinger and H. Diamond; Mechanical Engineering (New York, N. Y.), Vol. 51, p. 512; July, 1929.

"Applying the visual double-modulation type radio range to the airways." H. Diamond; *Bureau of Standards Journal of Research*, Vol. 4, No. 2, p. 287; Feb., 1930; and *Proceedings, Institute of Radio Engineers* (New York, N. Y.), Vol. 17, p. 2183; December, 1929.

"A tuned-reed course indicator for the 4 and 12 course radio range." F. W. Dunmore; *Bureau of Standards Journal of Research*, Vol. 4, No. 4, p. 473; April, 1930; and *Proceedings, Institute of Radio Engineers* (New York, N. Y.), Vol. 18, p. 980; June, 1930.

The deviometer is essentially a device for changing the relative sensitivity of the two reeds of the vibrating reed indicator, thereby permitting the pilot to fly courses (with equal reed deflections) along a line other than the equisignal line or zone set up by the beacon. In its simplest form the deviometer consists of a resistor shunted across the coils actuating one of the two reeds, thereby reducing the driving current through the actuating coils and consequently the sensitivity of that reed. The course, as determined by equality of reed deflections, is therefore shifted from the equisignal

* Communicated by the Director.

zone in the direction of the shunted reed. The same effect is thus accomplished on the airplane as by shifting the course at the beacon. In its more advanced form, the deviometer consists of a rheostat connected to the reed driving coils so as to vary the relative shunting effect on the reed driving coils. A movement of the pointer to the right or left reduces the shunting resistance across one or the other pair of reed driving coils and shifts the course obtained to the right or left of the equisignal zone set up by the beacon. The scale over which the pointer moves may be calibrated in degrees off the equisignal course and will be correct for all beacons having similar space characteristics.

The utility of the deviometer may be illustrated by the following examples. A pilot operating over a route between two airports, one of which has a radio range beacon but the other lying 15 degrees to the left of the radio-served airway simply adjusts the deviometer so that equal reed deflections are obtained along a line 15 degrees to the left of the true beacon course, and thereby lays out a new beacon course which he may use. The research division has made a number of tests on the deviometer in its various stages of development since this work was undertaken early last year and has within the past month lent one of these instruments to an air transport company for service tests.

Again, on a congested radio-served airway it becomes possible to reduce the hazards of collision between aircraft during conditions of poor visibility, by setting the deviometer to, say, 2 degrees to the right of the equisignal beacon course. Traffic in both directions will therefore fly on opposite sides of the equisignal course, and any possibility of head-on collisions is thus precluded.

A third use for the deviometer is its aid to a pilot who has deviated considerably to the right or left of the true course in order to avoid a storm area or for other reasons. A much more satisfactory beacon service may then be obtained by adjusting the deviometer so that the reed deflections are equal and flying a straight course to the airport rather than by trying to get back to the original beacon course.

It has been suggested that the deviometer may serve as a position finder and thus be useful to aircraft not flying a

beacon course. A pilot can tune in a range beacon station and adjust the deviometer which then gives him his bearing with respect to the equisignal course marked out by the beacon. Repeating this operation with another range beacon station gives a second bearing, and from the two a definite fix of the position of the aircraft is readily obtained. This possibility has not yet been tested out and it remains to be determined whether the direction discrimination of the system will be sufficient to make this particular application of the deviometer practicable.

Tests have indicated that the deviometer may be advantageously employed up to 15 degrees on either side of the equisignal zone. Beyond these limits, the useful distance range of the beacon is reduced, because of the shunting effect of the deviometer; and, in addition, the off-course indications become less accurate. A second limitation of the deviometer is the fact that the space characteristics of all beacons are not identical. This is particularly true for beacons the courses of which have been adjusted to fit several airways converging at arbitrary angles upon a given airport. However, when employed with these limitations in mind, considerable flexibility is added to the use of a radio range beacon normally giving but four beacon courses.

AN IMPROVED METHOD OF ULTRAVIOLET RADIOMETRY.

Since last February an improved method of ultraviolet radiometry has been under investigation. It is an attempt to solve the problem of measuring, in energy units, the ultraviolet from an extended source (e.g., carbon arc and reflector) for which purpose a suitable selective radiometer is required, but none is available, and for which purpose a spectroradiometer is unsuitable, because it analyzes the radiation emanating from only a small portion of the source.

The present procedure is: (1) to mount a thermopile and a special photoelectric cell, side by side, on a spectropyrheliometer (B. S. Sci. Paper No. 378); (2) isolate the ultraviolet radiation of wave-lengths 290 to 313 millimicrons; (3) measure the radiation intensity of wave-lengths 290 to 313 by means of a photoelectric cell, covered with a rectangular slit that

is of sufficient width to include these wave-lengths; (4) and similarly measure the radiation in this spectral range by means of a thermopile, covered with a slit that is cut into a template having the shape of the spectral response curve of the photoelectric cell.

By calibrating the thermopile against a standard of radiation a similar calibration of the photoelectric cell is obtained in absolute measure. After that, the photoelectric cell, in its holder, may be exposed to the source under investigation.

The ideal photoelectric cell for this purpose is one that does not respond to wave-lengths longer than about 315 millimicrons. During the past month, a photoelectric cell having closely these characteristics was investigated. Its sensitive surface consists of uranium. The cell is the invention of H. C. Rentschler, Director of the Westinghouse Research Laboratory. By using a uranium photoelectric cell in connection with the above mentioned apparatus, it is hoped to simplify the problem of ultraviolet radiometry.

SOLARIZATION OF SPECIAL WINDOW GLASSES.

In Technical News Bulletin No. 160 (August, 1930) attention was called to the fact that, after causing a decrease in transmission by exposure to sunlight, further exposure of the glass to sunlight of wave-lengths in the region of 365 to 400 millimicrons produces the greatest rejuvenation in transmission. On reëxposure of the glass to total sunlight, the transmission is again decreased; and this is recovered on further exposure of the glass to wave-lengths 365 to 400 millimicrons.

Concerning methods of accelerating the stabilization of the transmission of special window glasses by exposure to artificial sources and to the sun (solarization) it is significant to note the following facts:

(1) The use (at the start) of the quartz mercury arc to accelerate the stabilization has served the purpose of classifying the relative transparency of the various makes of window glasses to ultraviolet radiation;

(2) The long solarization tests, conducted during the past three years, by exposure of duplicate samples of these glasses to the sun, at sea level and at high elevations, have not changed this classification;

(3) Small variations (improvements) in the stabilized transparency of different melts of the same kind of glass can be determined readily in five to seven hours exposure to the lamp, instead of waiting an indefinite time for an uncertain total number of hours of sunshine, preferably summer sunshine;

(4) The use of a filter in front of the mercury arc may leave an uncertainty in the effect produced. In some cases the stabilized transmission obtained by exposure of the glass to the filtered radiation from the mercury arc is not so low as that obtained by direct sunlight, which defeats the purpose of the accelerated test. Since nearly all filters are affected by the ultraviolet rays of the lamp, a new filter would be required for each test, which does not seem practicable.

The following table gives the transmission in per cent. of various glasses at 302 millimicrons, when new and after exposure at a distance of 15 cm. from a 110-volt horizontal Uviarc, quartz mercury lamp for 10 hours; and of duplicate samples exposed to the sun for 5 to 12 months. Average thickness of glasses 2.3 mm. For earlier and more complete data see *B. S. Journal of Research*, Vol. 3, No. 5, p. 629, November, 1929 (Research Paper No. 113, price 15c).

Trade Name.	Number of Samples Tested.	Average Transmission.		
		New, Per Cent.	After Exposure to Lamp, Per Cent.	After Exposure to Sun, Per Cent.
Quartz glass	1	92	92	92
Corex D	3	61	59	60
Neuglas	3	63	50	57
Uviol-Jena	3	67	48	53
Helioglass	8	64	45	53
Sunlit	3	71 [1]	42	51
Vitaglass	20	58	33	42

In view of the fact that there is no assurance on the part of the distributor that the ultraviolet transmission of the glass, when stabilized, will not fall below a specified minimum

[1] High initial transparency may be produced by heat treatment, and is not indicative of the stabilized value.

value, the fairest procedure is to report averages on glasses examined during a period of 8 to 12 months.

The data obtained prior to February 1, 1929, are given in Research Paper No. 113 mentioned above. The data obtained on various makes of special window glasses examined during the 12 months prior to May 1, 1930, are given in the preceding table.

RUBBER VULCANIZED WITH TRINITROBENZENE.

Ordinary vulcanized rubber quickly tarnishes copper, silver, and mercury by reason of its sulphur content. However, when 1, 3, 5-trinitrobenzene is used as the vulcanizing agent instead of sulphur, rubber is obtained which has no apparent action on the above metals. Rubber tubing made with trinitrobenzene has been used in contact with mercury for several months with only slight fouling of the metal. When trinitrobenzene rubber is vulcanized in contact with copper, not only does the metal remain bright, but the aging qualities of the rubber are practically unaffected, quite contrary to the result obtained with sulphur vulcanized rubber under similar circumstances.

In order to ascertain quantitatively the extent if any to which copper was attacked by trinitrobenzene rubber, fine wires of known resistance were embedded in the rubber prior to vulcanization. Measurements of resistance made after vulcanization showed a decrease in resistance of about 0.1 per cent., which was probably due to the annealing of the wire. A comparison sample of copper wire, which was vulcanized in contact with a rubber compound containing a low percentage of sulphur and an ultra-accelerator, increased in resistance by 8 per cent. This indicates clearly that any action of trinitrobenzene rubber on copper is negligible in comparison to the effect of the most favorable sulphur-vulcanized rubber.

Trinitrobenzene rubber was also vulcanized in contact with silver, nickel, iron, manganin, advance, and nickel silver. In no case was there tarnish or other indication of attack on the metal.

The trinitrobenzene rubber used in these experiments was made according to the following very simple formula:

Pale crepe rubber	100 parts
Zinc oxide, "Kadox" brand	100 parts
1, 3, 5-Trinitrobenzene	2 parts
	202 parts

The time of vulcanization employed was 60 min. at 141° C.

Trinitrobenzene rubber may find an important application in the manufacture of electrical insulation by reason of the fact that it may be applied directly to copper without the necessity of tinning the latter. In view of this possible use, the electrical properties of trinitrobenzene rubber, made according to the above formula, were determined. The dielectric constant and power factor at 1000 cycles, 27° C., were 3.84 and 0.006, respectively. These are approximately the same as for sulphur-vulcanized rubber containing the same proportion of zinc oxide. The resistivity was 1.4×10^{15} ohm cm. which is somewhat less than for rubber vulcanized with sulphur, but adequate for practical insulation purposes.

NON-REDUCING DISACCHARIDES IN HYDROLYZED INULIN.

A paper which will be published in the November number of the *Bureau of Standards Journal of Research* discusses the constant occurrence of non-reducing disaccharides in hydrolyzed inulin.

A previous investigation showed that hydrolyzed inulin contained 92.8 per cent. of fructose, 3 per cent. of glucose, and 5.2 per cent. of a group of non-reducing disaccharides. The last may be an integral part of each molecule of inulin, in which case the molecular weight of inulin would be very great, or if a low molecular weight is to be ascribed to inulin the disaccharides must by arithmetical reasoning be a constituent of some molecules and not of others. If inulin is the heterogeneous mixture suggested by the second possibility, the ratio of molecules containing the disaccharides to those which are free from them would be expected to vary as the source of the inulin is varied, since in general the ratios of the different polysaccharides to each other vary greatly in different inulin-bearing plants. The problem then is to determine whether inulin derived from various plants contains

a constant or a variable percentage of the refractory disaccharides.

Samples of inulin were therefore prepared from a number of plants and carefully purified by recrystallization. Upon analysis the following mean percentages of non-reducing substance were found in hydrolyzed inulin from the following plants: wild chicory, 5.1; burdock, 5.7; dandelion, 4.9; golden rod, 5.3; Jerusalem artichoke, 5.4. The mean non-reducing content is 5.3 per cent., whereas Jackson and Goergen found in dahlia inulin 5.2 per cent. The specific rotation of this non-reducing substance was found by computation to be + 49, whereas the substance from dahlia inulin yielded the value + 55. It is therefore concluded that the non-reducing substance occurring in hydrolyzed inulin is the same in nature and in amount, quite regardless of the source from which the inulin is derived.

If the second possibility suggested above were true it should be possible to isolate by fractional crystallization portions of inulin in which the molecules containing the disaccharides have been removed or isolated, since some difference in solubility must accompany a difference in constitution. Samples of inulin from dahlias and from artichokes, however, after eleven fractionations showed practically an unaltered composition.

In order to determine whether the disaccharides were formed by side reactions during hydrolysis, related polysaccharides of higher solubility than inulin were hydrolyzed and analyzed. No evidence of the formation of the disaccharides was found.

It is concluded that the experiments can only be interpreted by the first possibility suggested above. Every molecule of inulin, regardless of its source, contains about 5 per cent. of the refractory disaccharides. Evidence is at hand that there are no less than three different disaccharides in the non-reducing mixture. The molecular weight of inulin calculated by assuming that three disaccharides are 5.3 per cent. of the whole molecule is found to be no less than 18,000. It is not the purpose of the bureau's report to emphasize the quantitative interpretation, but merely to indicate that the molecular weight of inulin is not that of six or nine hexose

residues as is commonly supposed, but is at least that of 40 and possibly as much as 120 hexose residues.

STARCH SIZING OF HIGHLY-PURIFIED WOOD FIBER PAPERS.

In an investigation in progress at the bureau, the paper-making qualities of highly-purified wood fibers are being studied by preparing from them, under semi-commercial mill conditions, the usual types of commercial papers. As a part of this investigation, a number of starch-sized papers were made to determine the effect of this sizing material on the papers. The amounts of starch employed and the methods of procedure in sizing were guided by the available information on the average commercial practice. Since the chief interest in this particular investigation is in the preparation of typical papers for testing in reference to the paper-making quality of the fiber under study, there was no attempt to make a thorough study of starch sizing as such. Results on strength increases due to sizing must therefore be considered as indicative only.

In the case of beater sizing with starch, the factors varied were acidity and percentage of rosin. Control samples were also prepared containing neither starch nor rosin. All papers were tested for: (1) strength, (2) chemical properties, (3) stability to a heat test of 100° C. for 72 hours. The testing procedures have been described in a bureau publication. It was found that in most cases approximately one-half the added starch was retained in the paper regardless of the presence or absence of rosin, or the acidity of the stock. The folding endurance and bursting strength of the papers were increased noticeably, the increases averaging about 15 per cent. Beater sizing with rosin resulted in strength decreases in all cases. The results of the aging tests indicate that the starch-sized papers had about the same stability as control sheets containing no starch, and thus that starch used in this way has little if any effect upon stability.

A number of surface-sized papers were also prepared. Three basic papers of purified-wood fibers were surface sized. These were: (1) an unsized paper, (2) a paper sized with one-half per cent. rosin, (3) a paper sized with one per cent.

rosin. Each of these basic papers was sized with the following: (1) glue, (2) glue and alum, (3) starch, (4) starch and alum. In the case of the starch-sized papers, the amount of starch retained in the sheet was found to be in proportion to the absorbency of the basic paper. The paper containing no rosin size retained about two per cent. starch, the paper sized with one-half per cent. rosin retained about one per cent. starch, and the paper sized with one per cent. rosin retained about three-fourths per cent. starch. The effect of surface sizing with starch on the strength of the papers was approximately as follows. The folding endurance was increased on the average by about 10 per cent. The bursting and tensile strength increases on surface sizing were as follows: the unsized basic papers, about 30 per cent.; the paper sized with one-half per cent. rosin, about 20 per cent.; and that sized with one per cent. rosin, approximately 15 per cent. The tearing strength alone underwent a decrease on sizing, and the decrease was roughly in proportion to the amount of starch taken up.

All the surface-sized papers, like the beater-sized papers, were found to be very stable toward the heat test, exhibiting very little change in strength and in chemical properties. In all cases upwards of 80 per cent. of the initial folding endurance was retained after heating, and the maximum decrease in alpha cellulose content was 1.5 per cent. The papers that were surface sized with starch containing alum (which was present to the amount of two per cent. of the weight of the starch) were found to be no less stable than the corresponding papers containing no alum in the size bath, which is further evidence that acidity introduced with surface sizing is relatively harmless if kept at a minimum.

In all the above-mentioned work the starches employed were prepared by non-oxidizing processes. Other experiments are in progress to determine the effects of oxidized starches.

THICKNESS OF SURFACE LAYER OF LIQUIDS.

The surface layer of any liquid possesses properties which are different in magnitude from the corresponding properties of the liquid in bulk. For example, the density and the vis-

cosity of the surface layer are probably somewhat greater than for the liquid as a whole. The thickness of this surface layer, in which the properties are different from the properties in bulk, is a question of theoretical and practical importance. The question is of theoretical importance because its answer will throw some light upon the distance to which one molecule may exert an appreciable attractive force upon its neighbors. It is of practical importance in special phases of the problem of lubrication.

Certain investigators in measuring the surface tension of soap solutions found evidence for the belief that the surface layer of the liquids was many thousands of molecules thick. A consideration of results of other methods of investigation leads to the view that the layer should not be thicker than perhaps one or two molecules. The work which purports to demonstrate the existence of a very thick surface layer is open to the criticism that the tests were made while the solutions were exposed to the air. Carbon dioxide is known to react chemically with soap molecules and the substance in the thick film may have been formed by this reaction.

In a paper to be published in the October number of the *Bureau of Standards Journal of Research*, surface tension tests are described which were made on soap solutions in an atmosphere free from carbon dioxide. These tests do not support the conclusion that the surface layer is of extraordinary thickness.

DETERIORATION OF WINDOW GLASS IN STORAGE.

A study has been undertaken to determine, if possible, the conditions necessary to prevent the scumming or surface deterioration of window glass in storage.

The samples used were furnished by six manufacturers and included both flat drawn and cylinder glass. Accelerated tests, to compare the resistance of the samples, were made by immersing the glass in water and solutions of sodium carbonate, trisodium phosphate, hydrochloric acid and iodeosin, and by subjecting the glass to steam in an autoclave. Storage tests were made by placing the glass over water in a closed container and included tests with the glass sheets in contact with each other and with the sheets separated by paper.

Only the autoclave tests gave results that were comparable to the results obtained by the storage tests.

Dipping the glass in dilute hydrochloric acid, a practice which is generally followed, seems to reduce weathering materially. Separating the sheets with a soft porous paper which contained very little mineral salts was also very beneficial.

The bulk of the evidence at the present time indicates that any of the glass used in this investigation will not deteriorate if stored so that moisture cannot condense on the surface of the glass. Since this condition cannot always be obtained, the use of an acid dip and packing paper seems to be desirable.

ELASTIC PROPERTIES OF CONCRETE.

In coöperation with the Aluminum Company of America, extensive measurements have been made on eight 24 by 48 inch concrete cylinders to determine the relations between loads and deformations.

The cylinders were made from concrete being placed during the construction of the Calderwood Dam in Tennessee. The concrete differed in composition and consistency from that ordinarily used for other types of construction. All the aggregates were made by crushing a hard type of arkose quarried at the site, the concrete containing one part Portland cement, three parts stone of sizes 0 to $\frac{3}{8}$-inch, and three parts stone of sizes 3 to 6 inch by loose volumes. An unusually "dry" consistency was used, the consolidation both in the dam and in the cylinders being effected by means of specially constructed motor driven vibrators.

The test cylinders were made during the months of June and July, 1929. They were kept damp for the first 14 days, then allowed to weather for about four months after which they were shipped to the bureau for testing. The specimens were from four to five months old when tested.

Both the longitudinal and lateral deformations produced by compressive loads equivalent to a stress range from 0 to 1400 lbs./in.^{2} were measured at five different locations.

It was found that the longitudinal deformations were proportional to the loads over a stress range from 0 to over 800 lbs./in.^{2} during the first loading and over the entire range

from 0 to 1400 lbs./in.^{2} for subsequent loadings. Readings taken at the low stresses of 20, 50, 100, and 200 lbs./in.^{2} indicated a linear relation between stress and strain, the same as for the higher stresses within the range considered.

No measurements of lateral deformations were made during the first loading, but for repeated loadings a linear relation was found between loads and lateral deformations over the entire stress range of from 0 to 1400 lbs./in.^{2} The average values for strength and elastic constants were: 3900 lbs./in.^{2} for compressive strength, 3,800,000 lbs./in.^{2} for modulus of elasticity and 0.15 for Poisson's ratio.

SOUND ABSORPTION COEFFICIENTS OF THE MORE COMMON MATERIALS.

The following figures have recently been obtained at the Bureau of Standards for the sound absorption coefficients of a number of materials now on the market as acoustic correctives. Figures are also given for the absorption of an audience seated in chairs of different kinds. The results have all been obtained by the reverberation method.

It is not necessarily the case that the materials of highest coefficient are the most advantageous. When there is room enough to apply the requisite quantity, a material of low coefficient will give the same result.

Material.	Absorption Coefficients for Frequencies.				
	128	256	512	1024	2048
Thermatex	.30	.39	.34	.43	.53
Acoustolith tile:					
Grade B, 1" thick	.10	.14	.28	.65	.73
Grade C, 1½" thick	.12	.19	.44	.61	.66
Grade C, 2" thick	.19	.26	.53	.64	.70
Armstrong cork tile, 1½" thick	.05	.09	.32	.49	.46

The coefficients given represent the fractional part of the energy of a sound wave which is absorbed at each reflection.

Some Investigations into the Velocity of Sound at Ultra-Sonic Frequencies Using Quartz Oscillators. C. D. REED. (*Phys. Rev.*, April 1, 1930.) The velocity of sound in air, both dry and with measured relative humidity, has been determined for frequencies ranging from 40 to 216 kilocycles. The air waves were set up by quartz oscillators actuated by a vacuum tube circuit. The absolute frequency of the oscillations was obtained by comparing the various quartz oscillators with a standard quartz crystal calibrated by reference to the clock at the Harvard Observatory. Waves from the oscillating quartz entered a space whose diameter was equal to 60 wavelengths for the lowest frequency used. The conditions were therefore quite different from those imposed by using a narrow tube. The waves were reflected from a plane surface parallel to the face of the emitting crystal, that could be moved forward and backward by a screw. The direct and the reflected waves interfered and formed a series of standing waves. The reaction of the reflected waves on the crystal causes a variation in the plate current of the actuating electrical system. "This current goes through a series of maximum and minimum values as the reflector is moved. The distance between successive maximum values of the current represents a half wavelength of the emitted sound." For dry air at 0° C. free from CO_2 the following velocities were obtained from readings taken over a distance of about 15 cm. located 45 cm. from the source:

Frequency kc./sec.	140	105	84	70	60	42
Vel./in m./sec.	331.60	331.63	331.74	331.63	331.73	331.75

The velocity in air containing water vapor was likewise measured. The emitting crystals would oscillate for only a few minutes in the damp air. For moist air at 20° C. the velocity is found by adding to the velocity of dry air at that temperature .14 of the relative humidity.

Although the difference is not more than about 1.6 m./sec. it remains true that the velocity is greater close to the source than farther away from it. The reason is not known.

G. F. S.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, OCTOBER 15, 1930.

The regular monthly meeting of The Franklin Institute was called to order by the President, Mr. Nathan Hayward, at which time the meeting was crowded to capacity.

The President immediately called upon the Secretary for business which had to be considered at the meeting. The latter reported that the minutes of the last meeting, that of Medal Day, had been printed in full in the June number of the Journal of the Institute. He moved that these minutes be approved as printed. The motion was seconded and adopted unanimously.

The Secretary then made a report of the changes in membership which had occurred since the last meeting: Two members were transferred from Resident to Life Membership; thirteen members were elected to Resident Membership; twelve to Non-Resident Membership; two new Student Members; and the death of ten members occurred.

The Secretary further stated that at this time various proposed amendments to the By-laws of the Institute were to be considered. All necessary notices of these changes had been given in due time and all the proposed changes had been approved by the Board of Managers. He then read the following proposed changes, with as much of explanation as was necessary:

ARTICLE IV.—Election of Officers.

Section 1. The officers shall be a President, not more than five Vice-Presidents, a Secretary, a Director of The Benjamin Franklin Memorial and The Franklin Institute Museum, a Treasurer, an Assistant Treasurer, a Controller, a Librarian, and twenty-four Managers.

Section 2. At the annual meeting of the Institute the President, the Vice-Presidents, and the Treasurer shall be elected to serve one year, and eight Managers shall be elected each year to serve for three years; provided, that the officers now elected, or who may hereafter be elected, shall continue to serve until their successors be elected.

Section 3. No change.

Section 4. Nominations for President, Vice-Presidents, Treasurer and Managers shall be made in writing at the stated meeting in the month of December. (No change in remainder of Section.)

Section 5. No change.

ARTICLE V.—Duties of Officers.

Section 1. No change.

Section 2. No change.

Section 3. No change.

Section 4. (NEW.) The Director of the Benjamin Franklin Memorial and The Franklin Institute Museum shall be appointed by the Board of Managers annually and shall have charge and supervision, subject to the Board of Managers, of the maintenance of the Memorial and Museum, of the exhibits therein displayed, and of the activities centering in the Museum. He shall report regularly, through the Secretary of the Institute to the Board of Managers, and may be appointed by the Board to membership in any of the committees of either the Board of Managers or of the Institute. The Secretary and Director shall recommend annually to the Board of Managers the appointment of the members of the scientific staff of the Museum.

Section 5. Formerly designated as Section 4. No other change.

Section 6. The Board of Managers shall appoint an Assistant Treasurer. He shall perform the duties of the Treasurer in the absence, or disability of the latter, or such duties as may be assigned to him.

Section 7. Formerly designated as Section 5. No other change.

Section 8. Formerly designated as Section 6. No other change.

ARTICLE VI.—Board of Managers.

Section 1. The Board of Managers shall have charge and control of the affairs of the Institute and of The Benjamin Franklin Memorial and The Franklin Institute Museum, and shall consist of twenty-four members elected as provided in Article IV. The President, the Vice-Presidents, the Secretary, the Treasurer and the Chairman of the Committee on Science and the Arts shall be ex-officio members.

Section 2. No change.

Section 3. No change.

Section 4. No change.

Section 5. No change.

Section 6. No change.

ARTICLE VII.—Committees of the Board of Managers.

The following standing committees of the Board of Managers, each consisting of five members, shall be appointed by the President and approved by the Board:

1. Executive. 2. On Finance. 3. On Membership. 4. On Publications. 5. On Bartol Research Foundation. 6. On Memorial and Museum.

The Executive Committee, etc. No change.

The Committee on Finance shall have charge of all securities and funds of the Institute. It shall invest all funds of the Institute available for investment and shall pay the income therefrom to the Treasurer of the Institute. It shall have control of the current funds of the Institute and be responsible for the preparation of the budget, in coöperation with the Controller. It shall have general supervision of all activities affecting the financial welfare of the Institute.

The Committee may, with the approval of the Board of Managers, appoint a Trust Company of the City of Philadelphia to act as Fiscal Agent under the direction of the Committee.

The Committee on Membership shall have charge of the membership of the Institute and shall report at each meeting of the Board of Managers any recommendations which it may see fit to make concerning membership.

The Committee on Publications shall have control of the publications of the Institute, subject to the approval of the Board of Managers.

The Committee on the Bartol Research Foundation shall have full charge, etc. No change.

The Committee on Memorial and Museum shall advise with the Secretary, the Director of the Memorial and Museum, and the scientific staff of the Museum concerning the activities and the exhibits of both the Memorial and the Museum.

ARTICLE VIII.—Board of Councillors.

Section 3. (NEW.) The Board of Councillors shall consist of not more than seventy-five members and shall embrace as ex-officio members, the Governor of the State of Pennsylvania, the Chief Justice of the Supreme Court of the State of Pennsylvania, the Mayor of the City of Philadelphia, the President of the City Council of Philadelphia, the President of the Board of Education and the Superintendent of Schools of the City of Philadelphia, the Provost and the President of the University of Pennsylvania, the Chairman of the Fairmount Park Commission, the President of the American Philosophical Society, the President of the Poor Richard Club, and the President and Secretary of The Franklin Institute. Additional members shall be nominated by the Board of Managers and elected by the Institute. The President of the Institute shall be Chairman of the Board of Councillors, and the Secretary of the Institute shall be Secretary of that Board.

It shall meet at least once a year, at which time the annual report of the President of the Institute shall be read. All questions of the policy and welfare of the Institute and the Museum may be presented to the meeting. In addition, the Secretary of the Institute shall send monthly to each member of the Board of Councillors, a concise statement of the activities, accessions, and accomplishments of the Institute.

Change former Article VIII to read, "ARTICLE IX."

Change former Article IX to read, "ARTICLE X."

Change former Article X to read, "ARTICLE XI."

Change former Article XI to read, "ARTICLE XII."

In Article X, formerly Article IX, Section 1, delete words "On Museum" and change number 4, to read, "3." In Section 2, delete "and Museum" and insert "and" between Library and Meetings.

It was then moved and seconded that the amendments proposed be approved. This motion was adopted unanimously and the By-laws were declared by the President to have been amended as proposed.

The Secretary then announced that Messrs. Cyrus H. K. Curtis, L. H. Kinnard, S. S. Fels and A. Atwater Kent had been elected to membership in the Board of Managers of The Franklin Institute, in order to fill vacancies caused by resignations or by an expansion of the Board, in accordance with the amendments of the By-laws just made.

Announcement was also made that Dr. James Barnes of Bryn Mawr College had been appointed to have charge of the Physics Section of the Museum; that Mr. Charles E. Bonine of the Engineering Firm of Bonine and Costa was to be in

control of the Engineering exhibits; and that Mr. James Stokley of Science Service, Washington, D. C., had been placed in charge of the Astronomical Section embracing the planetarium and the astronomical observatory of the Museum.

The Secretary announced also that others would be added to the staff as the necessity arose.

There was no further business so the President announced the paper of the evening and called upon Dr. Paul H. Dike, who spoke upon the theory of "Theremin."

Dr. Dike's discussion of the theory was followed by a demonstration of the use of the instrument by Mrs. Dike, a performer trained and licensed by the inventor. Dr. Dike's exposition and Mrs. Dike's demonstration constituted a unique feature in the life of the Institute. The combination gave the audience a most enjoyable evening and demonstrated clearly the purpose and the properties of "Theremin."

A vigorous discussion followed the presentation of Dr. and Mrs. Dike. The meeting was ended by a rising vote of hearty thanks, at nine-fifty p.m.

HOWARD MCCLENAHAN,
Secretary

October, 1930

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(*Stated Meeting, Board of Managers, October 15, 1930.*)

RESIDENT

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COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, October 8, 1930.)

HALL OF THE COMMITTEE,
PHILADELPHIA, OCTOBER 8, 1930.

MR. HENRY B. ALLEN *in the Chair.*

The following report was presented for final action:

No. 2925: Garrard Speed Changer.

This report recommended the award of the Edward Longstreth Medal to Mr. Charles G. Garrard, of London, England, for

"his invention of a means for the transmission of power as exemplified by the Garrard Speed Changer."

The following report was presented for first reading:

No. 2924: Process and Apparatus for Artificial Curing of Grains.

GEO. A. HOADLEY,
Secretary to Committee.

LIBRARY NOTES.

RECENT ADDITIONS.

Allen's Commercial Organic Analysis. Fifth edition. Volume 8. 1930.

American Electric Railway Engineering Association. Engineering Manual. 1930.

American Institute of Electrical Engineers. Transactions. Volume 49, No. 3. July 1930.

BARRETT, OTIS WARREN. The Tropical Crops. 1928.

Beilsteins Handbuch der organischen Chemie. Vierte Auflage. Erstes Ergänzungswerk. Fünfter Band. 1930.

CAMICHEL, CHARLES. Leçons sur les Conduites. 1930.

CASTELFRANCHI. GAETANO. Physique Moderne. Traduction française par M.A. Quemper de Lanascol. 1930.

DAVID, PIERRE. Manuel de reception Radio-Électrique. 1930.

Eastman Kodak Company. Elementary Photographic Chemistry. 1929.

FANSLER, P. E. House Heating With Oil Fuel. Third edition. 1927.

GEST, ALEXANDER PURVES. Engineering. 1930.

HERMANN, HUBERT. Techno-Diktionär: Deutsch-Englisch-Italienisch. 1929.

JONAS, KAREL. Slovník Česko-Anglicky s Úplnou Anglickou Výslovností. A Bohemian and English Dictionary. 1918.

Minerva Jahrbuch der gelehrten Welt. Dreissigster Jahrgang. Band 3. 1930.

MOUREK, V. E. Pocket Dictionary of the Bohemian and English Languages. 1920.

NAETHER, CARL A., and GEORGE FRANCIS RICHARDSON. A Course in English for Engineers. Volume 2. 1930.

Philadelphia City Directory. Volume 106. 1930.

Verein deutscher Ingenieure. Beiträge zur Geschichte der Technik und Industrie. Jahrbuch. Bd. 10–17. Eight volumes. 1920–1927.

WAGNER, PERCY A. The Platinum Deposits and Mines of South Africa. 1929.

Who's Who in America 1930–1931. Volume 16. 1930.

BOOK REVIEWS.

COMETS. By Charles P. Olivier, Ph.D., Professor of Astronomy and Director of the Flower Astronomical Observatory, University of Pennsylvania. 246 pages, 7 illustrations, 6 diagrams, 12mo. Baltimore, The Williams and Wilkins Company, 1930. Price $3.50.

It was in 1925 that Dr. Olivier, then at the University of Virginia, published "Meteors" (Williams and Wilkins Co.), the first modern book in English, and the only general work in any language since 1871, in this interesting field—a field in which he has been recognized as an international authority. The subject of comets has not been quite so neglected, for the last edition of George F. Chambers' "The Story of the Comets" appeared in 1910. But that work is now out of date in many respects, because of the developments in cometary astronomy in the last two decades. There has thus been a real need for a thoroughly scientific and modern work on the subject. Dr. Olivier's new book admirably meets this need.

In his preface, the author describes the scope of the book, his aim being, he says, to "cover briefly the history of the subject and the present theories of origin, constitution, changes and dissolution of comets," producing a work "useful to the astronomer who does not specialize in the subject, as well as to the average intelligent reader."

Further, he says, the book is intended to supplement, rather than to replace, the older works, such as Chambers', and is also intended as a sequel to "Meteors." This is necessitated not only because of the many close relations that exist between comets and meteors, but also because new developments and discoveries in the last five years have made possible an extension of some of the theories advanced in the closing chapters of the earlier work. Some of these theories were first publicly announced by Dr. Olivier in an address he gave before the Franklin Institute in January, 1929, and which was published in this JOURNAL in June, 1929.

In the first chapter he gives an interesting historical review of the subject. Then follows a chapter on general facts with regard to comets. In this he takes exception to the statement, often found in textbooks, that of the three main parts of a comet, *viz.*, the nucleus, the coma and the tail, it is the coma that must be present if the body is to be classed as a comet. It seems certain, he says, "that the coma could not exist without a nucleus, which would serve to give enough mass to hold the body together. . . . It seems necessary, therefore, to admit that all comets must have nuclei, whether we see them or not."

Some interesting facts are given regarding masses of comets. These are very difficult to determine accurately, but the fact that Lexell's comet of 1770 came within a million and a half miles of the earth without affecting the length of our year as much as a second, though its own period was shortened by 2½ days, shows that it must have had less than 1/13000 the mass of the earth. However, with the earth's mass some 6.59×10^{21} tons, a comet only a millionth of a millionth as massive would weigh 6.59×10^{9} tons, truly a tremendous figure, even though it would produce no observable perturbations.

The collision of such a mass with the earth would produce disastrous results

in the region where it hit, and Dr. Olivier considers in detail two cases where this has apparently happened. One is the famous "Meteor Crater" near Winslow, Arizona, where a meteoric mass struck the earth several thousand years ago and left the huge crater that is visible today to transcontinental airplane travelers. For many years this was thought to be of volcanic origin. Dr. Olivier gives credit to the late Dr. Daniel Moreau Barringer, a Philadelphia mining engineer and for many years a member of The Franklin Institute, as the originator of the meteoric impact theory of its formation. Though at first Dr. Barringer's ideas were violently opposed, they are now generally accepted, and have, in fact, been proven by the discovery of large amounts of meteoric material in the vicinity and buried deep in the ground.

Dr. Barringer supposed that the main mass of the celestial projectile lies buried near the crater, and devoted many years of an active life to trying to locate it. Dr. Olivier shares his view, but it should be pointed out that there is a strong dissenting opinion. At the recent Chicago meeting of the American Astronomical Society, Dr. F. R. Moulton expressed the opinion that the extremely high resistance that the ground would offer to such an enormous mass would completely demolish it, so that not a trace of the original material would be left. For a meteor of only a thousand tons, striking rock, Moulton estimated that a resistance of some 50,000 tons a square inch would be encountered. He further estimated that a close swarm of about 300,000 tons of meteors would have been required to produce the meteor crater. With such a mass the resistance would have been enormously higher, and so the original meteors would have been completely destroyed.

But regardless of whether or not the mass still remains, Dr. Olivier considers that it was really the head of a small comet. The only other seeming collision of the earth with a comet that has been studied in any detail was on June 30, 1908 when a meteoric mass fell in central Siberia. Dr. Olivier believes this also to have been the head of a small comet, perhaps a piece of the Pons-Winnecke comet that had broken off from the parent body and embarked upon a cometary career of its own. Support is lent this view by the fact that on the date mentioned the earth was only 0.03 astronomical units away from the Pons-Winnecke orbit, and that in 1916 this comet gave a fine little meteor shower.

Other chapters in the book deal with such phases of cometary astronomy as the tails of comets, their spectra, comet families, and several comets specifically. These are Halley's, most famous of all; Biela's, which broke into two and then disappeared as a meteor shower; Moorehouse's, which made such rapid changes; Pons-Winnecke's and 1910a. Of particular interest is the chapter on Halley's comet, for in 1910, the year of its last appearance, Dr. Olivier was at the Lick Observatory and assisted Dr. H. D. Curtis, who made perhaps the most extensive series of photographs of this comet. Except for Dr. Curtis himself, probably no one saw as much of it as Dr. Olivier, so much of this chapter is made up of personal impressions.

In the final chapter, on conclusions, we find perhaps the most important part of the book from a scientific standpoint. Here Dr. Olivier presents for the first time a new theory of the origin of comets and meteors. Accepting the planetesimal hypothesis of Moulton and Chamberlin, he assumes that a passing star produced two huge tidal bulges on the sun, one on the side towards it and

the other opposite, and that these finally broke off, producing the raw material for the planets. But, he argues, the material that heaped up in these bulges had to come from somewhere else on the sun, so at the same time there was a low tide belt at right angles to them. With surface material removed, deeper, hotter, and presumably more explosive matter was exposed. This would then be violently erupted all around this low tide zone. The material thrown out would be pulled by the gravitational attraction of the other star, but masses ejected on opposite sides of the low tide belt would be given motions in exactly opposite directions. Some would mingle with material torn from the other sun.

All this ejected material, suggests Dr. Olivier, formed the comets and meteors as well as many asteroids and perhaps some of the satellites. Such a theory easily explains the retrograde motions of many of these bodies and their varying, and in some cases high, inclinations to the ecliptic.

This book, therefore, is to be recommended both to the scientist who wants a complete summary of the essentials of cometary astronomy and to the layman who wants a very interesting and readable account of some of the most fascinating objects that come within reach of the astronomer's telescope and, occasionally, the naked eye. Though non-mathematical in the main text, an appendix gives a brief account of the elementary facts about orbits. Physically, the printing, the paper, and the binding all combine to make it a work of which the publishers may well be proud, and, except for a few unimportant and obvious typographical errors, it realizes their motto of "Sans Tache."

JAMES STOKLEY.

THE MARITIME PROVINCES, CANADA; being a short description of the developments in New Brunswick, Nova Scotia, and Prince Edward Island. 79 pages, illustrations, map, 12mo. Ottawa, National Development Bureau, Department of the Interior, n. d.

From this booklet the reader can in a few moments acquire much authoritative information about the progress and potentialities of New Brunswick, Nova Scotia and Prince Edward Island, their principal resources and development.

The wealth of these provinces has been overlooked during the growth of western Canada and the industrial expansion of Ontario and Quebec, and it is only now that they are receiving the recognition to which their resources in agricultural lands, forests and fur-bearing animals, minerals, fisheries and water-powers rightly entitle them.

Eight pages are devoted to Recreational Resources.

The work is illustrated by half-tone illustrations from original photographs.

R.

THE ADJUSTMENT OF ERRORS IN PRACTICAL SCIENCE. By R. W. M. Gibbs, B.A. (Camb.), B.Sc. (Lond.). 112 pages, 43 figures, 12mo. New York, Oxford University Press; London, Humphrey Milford, 1929.

This little volume represents an attempt to simplify and condense into a readable form the gist of the Theory of Errors. Primarily it is intended for the use of research students in practical science but various portions no doubt will be found to be of value to students of sociology, psychology and education. In these

three studies, all data must be subjected to a correlation process before worthy deductions can be drawn.

Following the introduction, the author discusses in turns, Probability Normal Curve of Errors, Area and Volume, Distribution, Line of Closest Fit, the Correlation Coefficient, Partial Correlation and Weighted Observations, Correlation Ratios and Errors. The appendix contains a list of twenty-six formulas.

T. K. CLEVELAND.

LA GAMME, INTRODUCTION A L'ETUDE DE LA MUSIQUE. Par P.-J. Richard, Anciene eleve de l'Ecole Polytechnique. Preface de M. Marius Casadesus. viii–231 pages, 12mo, paper. Paris, Librairie Scientifique Herman et Cie. Price 28 francs.

It is generally recognized that all art is susceptible of analysis in accordance with the principles of physical science provided all the component causes which enter into an effect pleasing to the senses are capable of classification for the purpose of formulation. In modern times, the purely physical features of tone have been the subject of a great amount of brilliant development, only to mention the classic investigations of Von Hemholtz, Lord Rayleigh and Koenig. More recently architectural acoustics and acoustical apparatus have also made important strides.

The particular theme of the present work is an examination of that graduated series of tones, known as the musical scale, on a purely physical basis, what rules govern the intervals between these tones and a consideration of the methods which have been adopted in actual practice in constructing such scales in accord with the artistic requirements of musical composition. The work is intended primarily for the young students of the *Conservatoire* and schools of music that they may be provided with an arithmetical and physical explanation of the musical scale independently of artistic considerations and instrumental technique. Dealing wholly with arithmetical relationships in the analysis, the author in the first chapter wisely provides a review of the arithmetic of fractional numbers, certain theorems of inequalities, and logarithms which are employed in the discourse. The author's procedure is first a presentation of the historic evolution of scales from early times and the numerical relations between their component tones, a consideration of their relation to musical acoustics and an analysis of scales which have received common adoption. All of these receive searching numerical comparison. Regarding the physico-mathematical derivation of a rational scale, the author says: "The scale is not a mathematical production. It is not expressed in rigorous figures which may not be varied without offending the ear. The ear, fortunately, though very sensitive, permits approximations; and these approximations, between which a choice may be made, are represented by numerical series which it is convenient to employ in the resolution of certain elementary problems of musical acoustics, and in arriving at an explanation of certain musical facts."

The work is a thoroughgoing study of the musical scale to which the historical notes interspersed throughout its rather formidable collection of numerical data, and the concise account of the physics of the subject supply a needful zest for the perusal of the volume.

L. E. PICOLET.

PUBLICATIONS RECEIVED.

Sheet Steel and Tin Plate, by R. W. Shannon, 285 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1930. Price $5.00.

Leonardo da Vinci the Anatomist, by J. Playfair McMurrich, 265 pages, illustrations, plates, quarto. Baltimore, The Williams and Wilkins Company, 1930. Price $6.00.

Etoiles et Atomes, par A. S. Eddington, traduction J. Rossignol, 188 pages, plates, 8vo. Paris, Hermann et Cie., 1930. Price 35 francs.

Canada, Department of the Interior, National Development Bureau, The Maritime Provinces, 79 pages, tables, illustrations, 12mo. Ottawa, Kings Printer, n. d.

Handbook of Technical Instruction for Wireless Telegraphists, by H. M. Dowsett, 487 pages, illustrations, 8vo. London, Iliffe & Sons Ltd., n. d. Price 28 sh.

The Elementary Principles of Wireless Telegraphy and Telephony, third edition, by R. D. Bangay, revised by O. F. Brown, M.A., 268 pages, illustrations, 8vo. London, Iliffe & Sons Ltd., n. d. Price 10 sh. 6 d.

Artificial Sunlight, Combining Radiation for Health with Light for Vision, by M. Luckiesh, 254 pages, illustrations, plates, tables, 8vo. New York, D. Van Nostrand Company, Inc., 1930. Price $3.75.

Comets, by Charles P. Olivier, 246 pages, illustrations, plates, 8vo. Baltimore, The Williams and Wilkins Company ,1930. Price $3.50.

CURRENT TOPICS.

History of the Determination of the Heights of Mountains. FLORIAN CAJORI. (Isis, Dec., 1929.) There are three methods of measuring the elevation of mountains above sea level or above adjacent plains, the geodetic, which is the most ancient and at the present time the most accurate, the barometric and finally, the boiling point method. Dicaearchus (350–290 B.C.), Eratosthenes, a contemporary of Archimedes, and Xenagoras are credited with having made the first measurements of mountain heights. Their estimates of heights were in error by amounts varying from 20 to 36 per cent. The method employed by them is nowhere stated but it probably depended on the comparison of similar triangles. The Greeks had five instruments capable of being used in measuring mountain heights, the dioptra, the quadrant, the ancient astrolabe, the ancient cross-staff and the gnomon. The author adduces reasons that eliminate all of these except the dioptra, which was capable of measuring heights by a series of levelings. It is certain that leveling was carried out with much precision, the evidence being the construction of the canal connecting the Nile with the Red Sea and the Mount Athos canal cut by Xerxes in 480 B.C.

For a stretch of 1800 years little progress was made in geodetic measurements. Cardan, the mathematician, in the sixteenth century put the greatest heights of mountains at 288 miles. It came to be recognized that atmospheric refraction had to be reckoned with in measurements of long range. Snell in Holland (1617) and Picard 55 years later in France introduced improved technique and better instruments. Months and years were seen to be required for the determinations for which hours and days had been regarded as adequate. In the last three centuries the technique has been much improved, though atmospheric refraction continues to reduce the accuracy of results.

The barometric method. Pascal in France came to the conclusion that the height of the mercury in a barometer tube should be less at a high altitude. After uncertain results had been obtained in a high church tower in Paris the experiment was repeated in Auvergne on September 19, 1648. Périer, Pascal's brother-in-law, prepared two equal barometer tubes, each four feet long. In both the height of the mercury column was 26 inches 3.5 lines. One instrument was kept at Clermont and observed at intervals through-

out the day, while the other was carried to the top of the mountain, Puy de Dôme, by Périer escorted by a party of clerics and laymen. On top the barometric reading was only 23 in. 2 lines—a reduction of 3 in. and 1.5 lines. At the foot of the mountain the reading of the movable barometer was observed to be the same as before the ascent. Within the last few months the top of this historic mountain has been sold at auction and purchased by some governmental unit. Pascal composed an account of his experiment but, though printed, it was withdrawn from circulation. It was not till 1651 that an available account appeared in an anatomical publication by Pecquet. Thus the scientific world became acquainted with Pascal's experiment. The Accademia del Cimento in Florence was incited to try further experiments with the barometer, which, however, introduced the complicating factor of air temperature. In Scotland George Sinclair from 1661 to 1666 repeated the experiment on three elevations of Scotland and in Glasgow cathedral, as well as in mines. He assumed the entirely false relation that the drop in barometric height is proportional to the distance through which the instrument is raised. Otto von Guericke tried to repeat the experiment in the Harz mountains but a servant fell and broke the barometer.

Many precautions must be taken before elevations can be accurately measured by barometer readings. Changes of temperature, capillary action, the presence of air in the mercury, all militate against reliable results. Halley in the Philosophical Transactions of 1686 gave a logarithmic formula for calculating elevations from changes in the height of the mercury column. Its form is correct but he was ignorant of the correct values of the constants involved. During the seventeenth century the barometer was not employed to measure elevations. The first to do this was the Swiss, J. J. Scheuchzer from 1705 to 1707. During the first half of the eighteenth century many discordant results were obtained in getting heights by barometric means. The correct formula was not always applied.

DuFay in 1723 suggested the desirability of boiling the mercury in the barometer tube, but this did not meet with general approval. Later it was recognized that the escape into the vacuum above the mercury of air or water vapor produced a depression of the mercury column and distorted the readings. More attention was given also to the effect of capillary depression. In the second half of the eighteenth century many were working to improve the barometer and Laplace followed by Gauss and Bessel developed formulas that took account of variations in gravity and differences in temperature and

humidity. As early as 1810 it was noticed that barometrically determined heights vary with the hour of day at which the measurement is made. In the U. S. Coast Survey Report for 1871 the difference in height between two places in California determined by leveling was 598.72 m., but by the barometric method at 7 A.M. it was 598.80, at 1 P.M. 610.34 and at 5 P.M. 604.32. These differences were attributed to "a variable difference in the observed mean temperature at the two stations and the actual average temperature of the intervening stratum of air."

Thermometric method. This depends on the lowering of the boiling point of a liquid by reason of reduced pressure. It does not furnish as accurate results as the barometric method.

G. F. S.

The Photometry of Mass Spectra and the Atomic Weights of Krypton, Xenon and Mercury. F. W. ASTON. (*Proc. Roy. Soc.*, A 802.) Experimental data on that attractive problem of physics, the evolution of matter, at the present time come chiefly from the group of radioactive elements. From the rest of the elements significant information is obtained by a study of the relative abundance of atoms of different types. This examination may include all matter accessible to analysis or, in a restricted field, it may ascertain the relative abundance of the isotopes of one chemical element after the other. The former way has been long followed but its results are of uncertain value. Is the earth a just sample of the universe? If it is, we must face the fact that a shell only a few miles in thickness is the sole portion directly known, while the vast interior is out of reach. "It has been pointed out that the scarcity of the inert gases is quite abnormal and suggests that even our planet as a whole may not be a fair sample after all."

"In contradistinction to this the second part of the problem is virtually free from the uncertainty of sampling. Since the discovery of isotopes among the inactive elements hundreds of analyses have been performed to test the unvarying nature of the atomic weights of naturally occurring complex elements, and, with the possible and numerically insignificant exception of boron, all have given consistently negative results. Only in the laboratory has variation in abundance of isotopes been achieved, and that with immense labour and to a very small degree. We may therefore say briefly that whereas it is impossible to obtain a mixture of elements which we may regard as a fair sample of cosmic matter it is very nearly as impossible to obtain a mixture of isotopes which is *not* a fair sample of a complex element. There is therefore some

reason for hoping that the study of the relative abundance of isotopes may yield results more directly applicable to the fundamental problem than that of the relative abundance of elements."

All odd numbered elements seem to have at most two isotopes. The relative abundance of these can be calculated from their mass numbers, the atomic weight of the complex element and the packing fraction, though for accurate results the chemical atomic weight needs to be known to the second decimal place. Aston has however devised an experimental method that is applicable to all elements having isotopes according to the evidence of the mass-spectrograph. It relies on measurement of the optical intensity of the lines recorded on the photographic plate by the constituent isotopes of a complex element. Great care was exercised in removing errors due to variation in the intensity of the discharge from which the isotopes were derived. Krypton possesses several properties advantageous for the successful application of this method and on this element it was tried. For the six isotopes the following values were obtained:

Krypton.

Mass Numbers of Isotopes	78	80	82	83	84	86
Relative Abundance, $Kr^{84} = 100$	0.74	4.31	20.75	20.75	100.	29.37
Percentage Abundance	0.42	2.45	11.79	11.79	56.85	16.70

From the above values the atomic weight of krypton is calculated to be 83.77. In 1901 Ramsay obtained 82 and in 1908 Moore got 83.012, while Aston himself in 1922 using the mass-spectrograph got 83.5.

The method was further applied to mercury and xenon with these results,—

Mercury.

Mass numbers of isotopes	196	198	199	200	201	202	204
Relative Abundance $Hg^{202} = 100$	0.34	33.8	56.2	81.2	46.7	100	23.4
Percentage Abundance	0.10	9.89	16.45	23.77	13.67	29.27	6.85

The calculated atomic weight of Hg is 200.62 in close agreement with 200.61 as given in the 1925 edition of the International Table of Atomic Weight.

Xenon.

Mass Number of Isotopes	124	126	128	129	130	131	132	134	136
Relative Abundance $Xe^{129} = 100$	0.28	0.28	8.5	100	15.4	76.2	97.5	38.0	32.4
Percentage Abundance	0.08	0.08	2.30	27.1	4.18	20.7	26.4	10.3	8.79

The calculated atomic weight of xenon comes out equal to 131.27. Ramsay in 1901 found it to be 128 and Moore in 1908, 130.70. Aston in 1922 got 131.3.

Much more work must be done on the relative abundance of isotopes of the same element before any general conclusions can be drawn, and Aston with his long experience and great industry will, no doubt, proceed to obtain data for other complex elements.

G. F. S.

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